1.5 Absolute Value

$|x| = 4$

1.5 Absolute Value

Learning Targets:

1. SWBAT define and evaluate an absolute expression of a single value.

2. SWBAT rewrite an absolute expression as a piecewise function thus removing the absolute value signs.

3. SWBAT solve absolute value equations .

4. SWBAT solve absolute value inequalities.

5. SWBAT graph the absolute value of a function, given the graph of the function.

Informal Definition

Absolute value is the **distance** a number is from zero on the number line.

$$|14| = |-14| = 14$$

The **absolute value** of a number can **never** be **negative!**

Ex.1 Find the absolute value of the following:

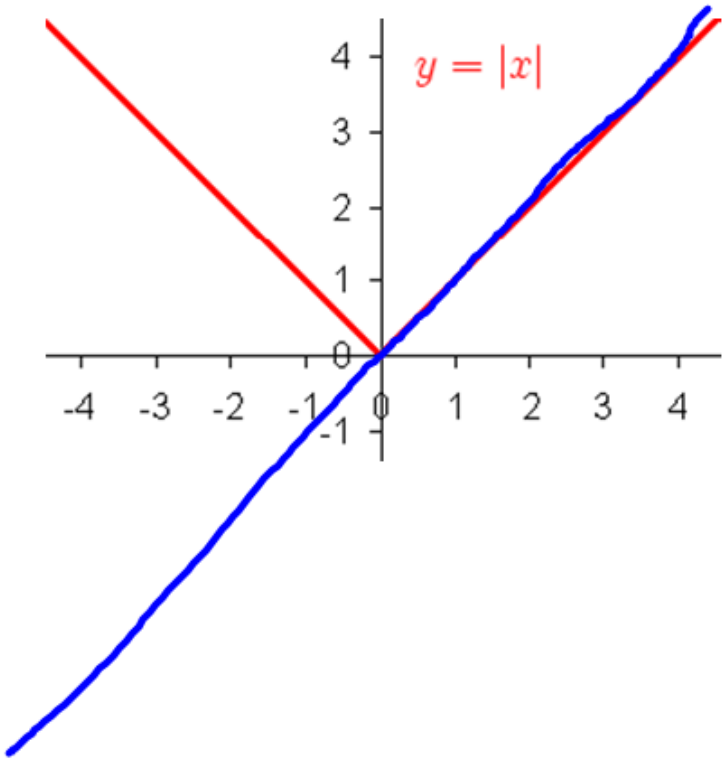
a) $|-23|$

b) $|-4\pi|$

Formal Definition

$y = |x|$

$$|x| = \begin{cases} x, \text{ if } x \geq 0 \\ -x, \text{ if } x < 0 \end{cases}$$

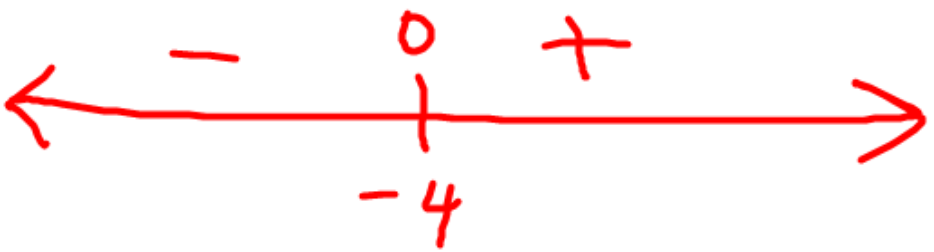


Removing Absolute Signs

Ex.3 Write an expression equivalent to the following without using absolute value signs.

a) $|x+4|$ $=$ $\begin{cases} x+4 & \text{if } x \in [-4, \infty) \\ -(x+4) & \text{if } x \in (-\infty, -4) \end{cases}$

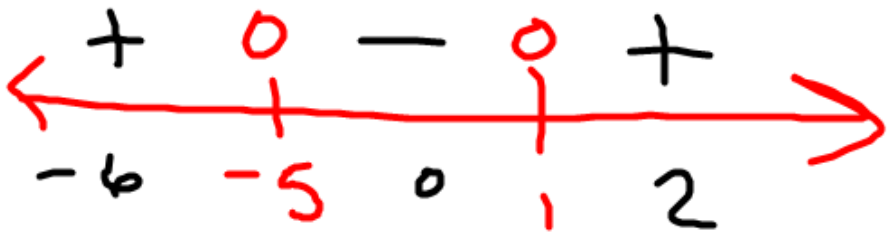
$x+4$



b) $\left|x^2+4x-5\right| =$

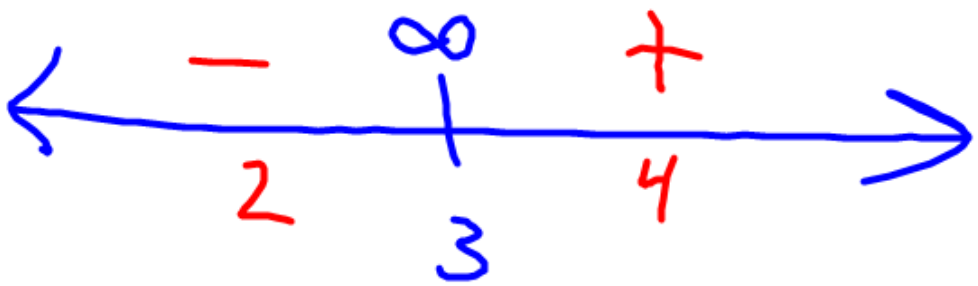
$$\begin{cases} x^2+4x-5 & \text{if } x \in (-\infty,-5] \cup [1,\infty) \\ -(x^2+4x-5) & \text{if } x \in (-5,1) \end{cases}$$

$(x+5)(x-1)$



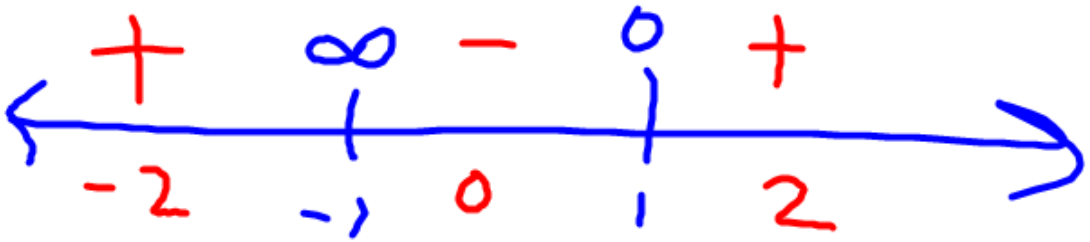
c) $\left|\frac{2}{x-3}\right| = \begin{cases} \frac{2}{x-3} & \text{if } x \in (3,\infty) \\ -\left(\frac{2}{x-3}\right) & \text{if } x \in (-\infty,3) \end{cases}$

$$\frac{2}{x-3}$$



d) $\left|\dfrac{x-1}{x+1}\right| = \begin{cases} \dfrac{x-1}{x+1} & \text{if } x \in (-\infty,-1) \cup [1,\infty) \\ -\left(\dfrac{x-1}{x+1}\right) & \text{if } x \in (-1,1) \end{cases}$

$$\frac{x-1}{x+1}$$



e) $\dfrac{2x}{|x|}$

Before absolute value therapy, Jerome was feeling rather negative.

After absolute value therapy, Jerome was feeling much more positive.

Solving Absolute Value Equations and Inequalities

Ex.1 Solve the following equations and verify:

a) $|9-5x| = 6$

$|x+4| - 7 = 4$

$|x+4| = 11$

Two Cases

$9-5x=6$ OR $-(9-5x)=6$

$-5x=-3$

$-9+5x=6$

$5x=15$

$x=3$

$x = \frac{3}{5}$

Verify

$|9-5(\frac{3}{5})| \stackrel{?}{=} 6$ ✓

$|9-5(3)| \stackrel{?}{=} 6$

$|-6| = 6$ ✓

b) $|15-2t| = t+6$

Ex.2 Solve the following inequalities. Write your solutions in interval notations.

a) $|-5x+4| \leq 11$

$[-7/5, 3]$

$-5x+4 \leq 11$ OR $-(-5x+4) \leq 11$

$-5x \leq 7$

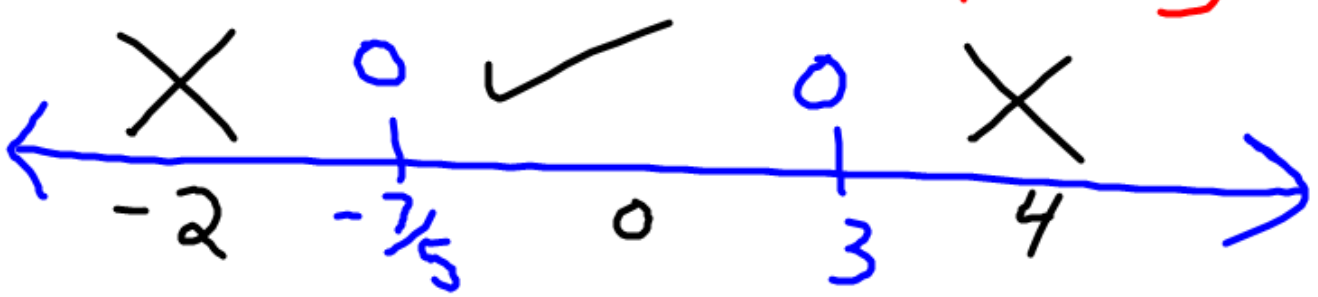
$x \geq \frac{7}{-5}$

OR

$5x-4 \leq 11$

$5x \leq 15$

$x \leq 3$



b) $\left|\frac{5}{x-4}\right| > 1$

$x \neq 4$

line test

$\frac{5}{x-4} > \frac{1}{1}$

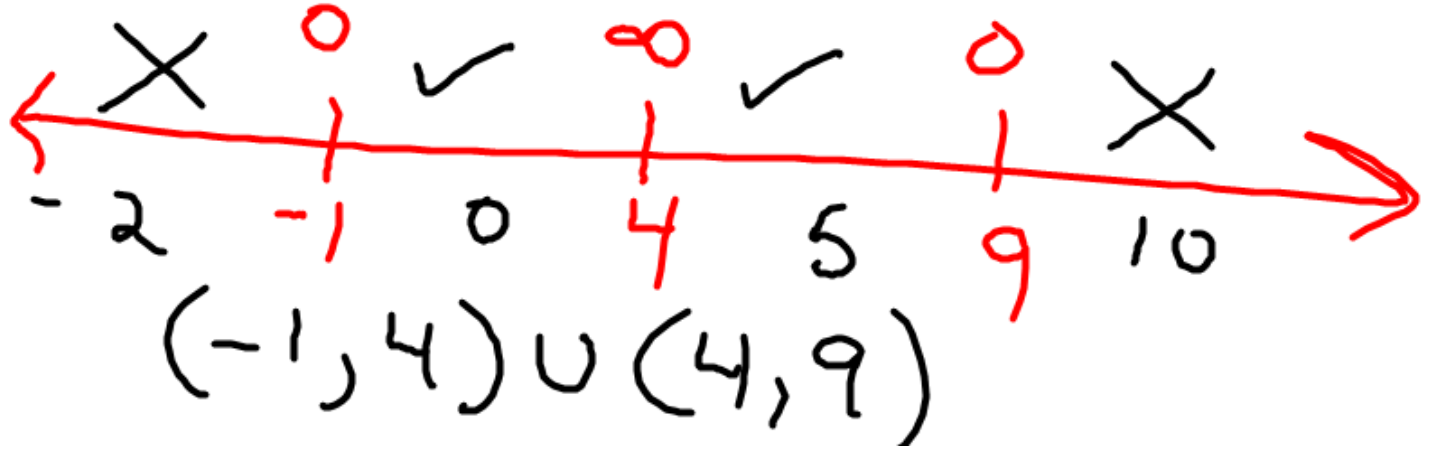
$5 > x-4$

$9 > x$

$\frac{-5}{x-4} > \frac{1}{1}$

$-5 > x-4$

$-1 > x$



$(-1, 4) \cup (4, 9)$

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#'s 17,19,21,24,25,26,29,30,43,44,45,49,51,53,54,57