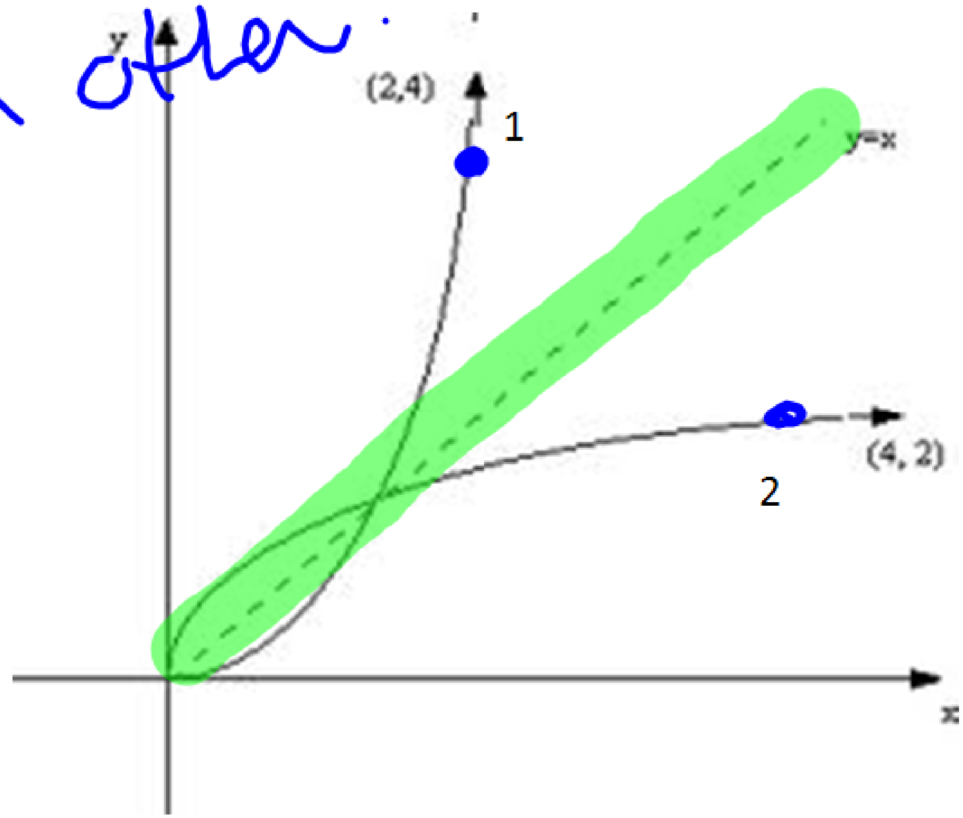
1.4 Inverse of a Relation

What are some things you notice about the graphs 1 and 2?

Graphs 1 and 2 are inverses of each other.

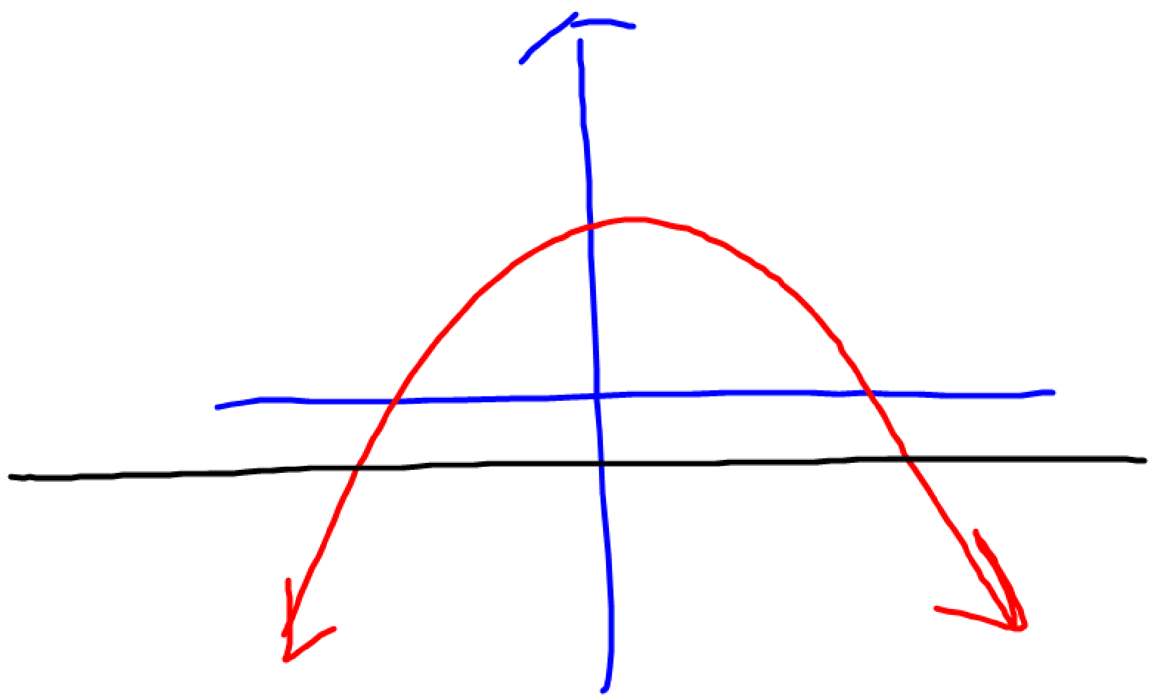


Graphs 1 and 2 are actually inverses of each other.

The inverse of a relation is found by interchanging the x-coordinates and y coordinates of the ordered pair of the relation.

The inverse of a function $y=f(x)$ may be written in the form $x=f(y)$.

The inverse of a function is not necessarily a function.



When the inverse of *f* is itself a function, it is denoted by f^{-1}, read as "*f inverse*".

The -1 in $f^{-1}(x)$ does not represent an exponent; that is

$$f^{-1}(x) \cancel{\neq \frac{1}{f(x)}}.$$

x^{-2}

$\frac{1}{x^2}$

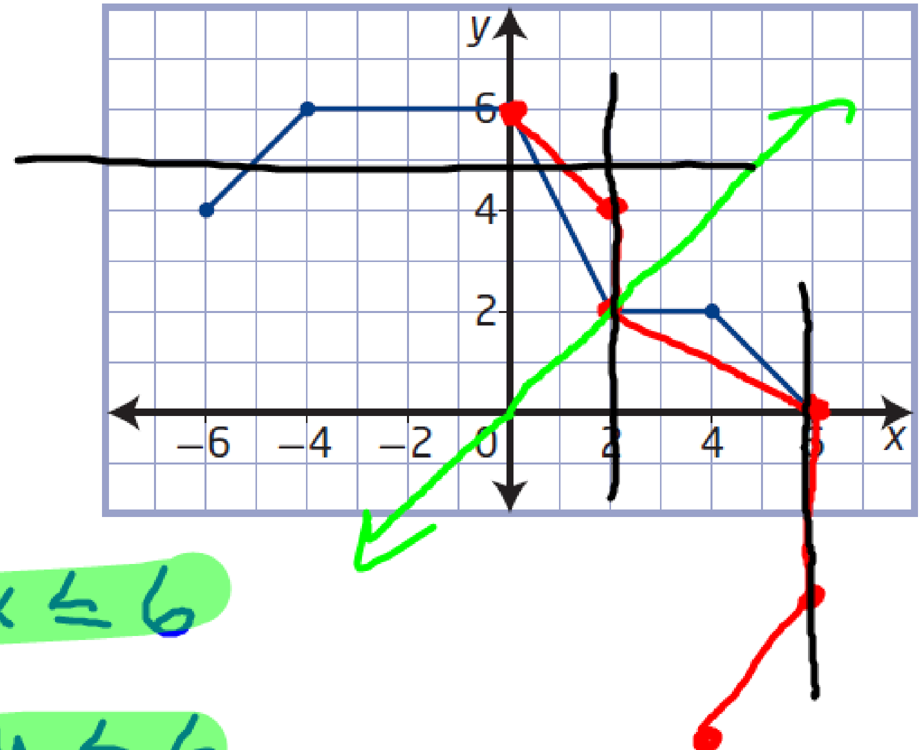
Drawing Graph of the Inverse Given the Graph of the Relation

Consider the graph of the relation shown.

a) Sketch the graph of the inverse relation.

b) State the domain and range of the relation and its inverse.

c) Determine whether the relation and its inverse are functions.



b) R → D: $-6 \le x \le 6$

R: $0 \le y \le 6$

Inverse → D: $0 \le x \le 6$

R: $-6 \le y \le 6$

c) R → f

I → not function

d) (2, 2)

Your Turn

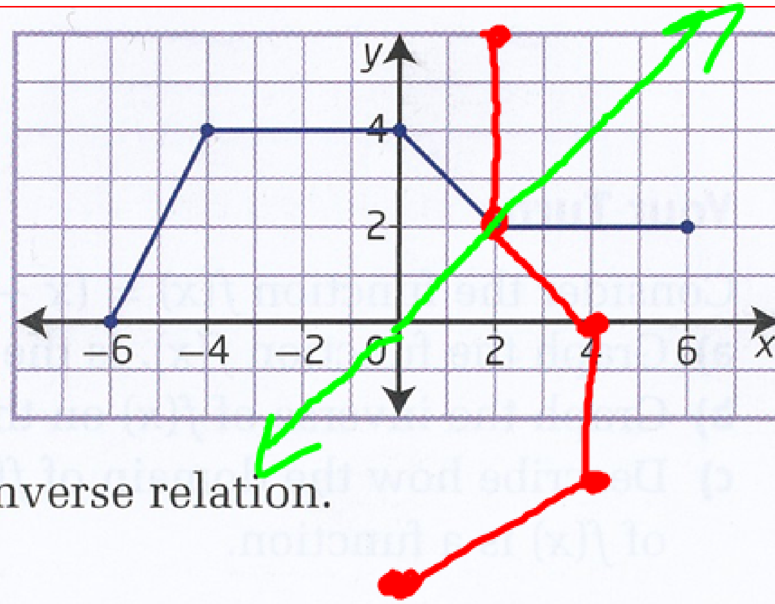
Consider the graph of the relation shown.

a) Determine whether the relation and its inverse are functions.

b) Sketch the graph of the inverse relation.

c) State the domain, range, and intercepts for the relation and the inverse relation.

d) State any invariant points.



a) R → function

I → Not a function

c) R → D: $-6 \le x \le 0$

R: $0 \le y \le 4$

x int (−6, 0)   y int (0, 4)

I → D: $0 \le x \le 4$

R: $-6 \le y \le 6$

x int (4, 0)   y int (0, −6)

c) D: $x \geq -2$ OR D: $x \leq -2$

Example 1

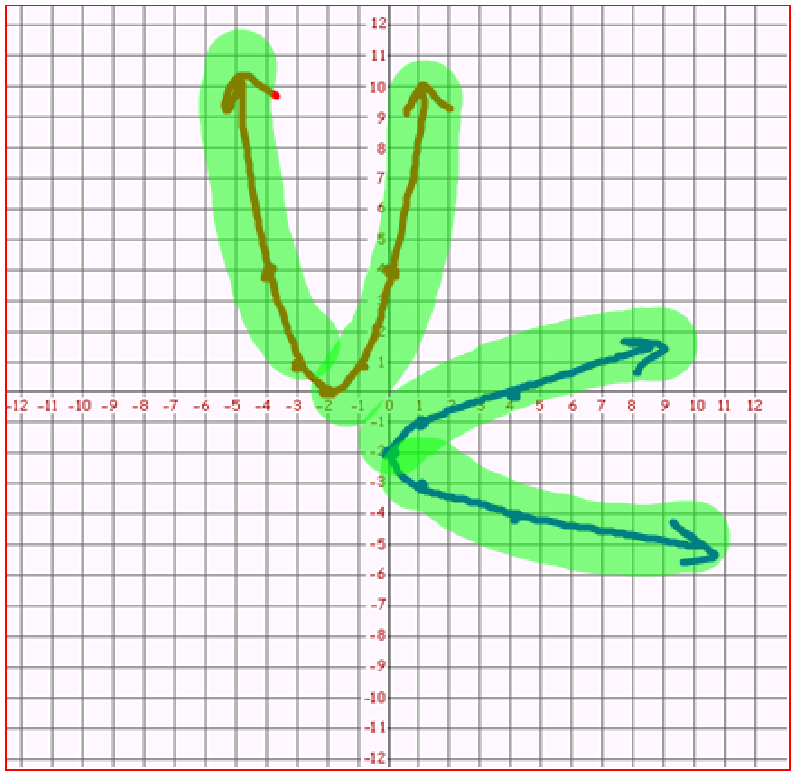
Consider the function $f(x) = (x + 2)^2$.

a) Graph the function $f(x)$. Is the inverse of $f(x)$ a function?

b) Graph the inverse of $f(x)$ on the same set of coordinate axes.

c) Describe how the domain of $f(x)$ could be restricted so that the inverse of $f(x)$ is a function.

x	y
-2	0
-1	1
-3	1
0	4
-4	4



Inverse

x	y
0	-2
1	-1
1	-3
4	0
4	-4

Determining the Equation of an Inverse.

To find the equation of the inverse of a relation you simply switch the *x* and *y* variables and solve for *y*.

Example 2

Determine the equation of the inverse $y = -3x + 7$.

$$y = -3x + 7$$

$$x = -3y + 7$$

$$\frac{3y}{3} = -\frac{x}{3} + \frac{7}{3}$$

$$y = -\frac{1}{3}x + \frac{7}{3}$$

$$= \frac{-x+7}{3}$$

Example 3

Determine the equation of the inverse $y = x^2 - 9$.

$$x = y^2 - 9$$

$$\sqrt{x+9} = \sqrt{y^2}$$

$$\boxed{\pm\sqrt{x+9} = y}$$

Key Ideas

- You can find the inverse of a relation by interchanging the x-coordinates and y-coordinates of the graph.
- The graph of the inverse of a relation is the graph of the relation reflected in the line $y = x$.
- The domain and range of a relation become the range and domain, respectively, of the inverse of the relation.
- Use the horizontal line test to determine if an inverse will be a function.
- You can create an inverse that is a function over a specified interval by restricting the domain of a function.
- When the inverse of a function $f(x)$ is itself a function, it is denoted by $f^{-1}(x)$.
- You can verify graphically whether two functions are inverses of each other.

Assignment Page 51

#'s 1,2,3,4,5b,c,f,

6,8,9,11,12c,f,14,15,20, 21