

1.3 Division By Zero

$$\frac{0}{7}$$

$$\frac{7}{0}$$

$$\frac{0}{0}$$

1.3 Division by Zero

Learning Targets:

1. SWBAT determine the values of x that make a rational expression zero and the graphical implications of these x values.
2. SWBAT determine the values of x that make a rational function undefined and the graphical implications of these x values.
3. SWBAT determine the values of x that make a rational function indeterminate and the graphical implications of these x values.



Meaningful Division

For division to be defined and have meaning, a given quotient must have only one result that checks. Thus

$\frac{100}{4} = 25$, has meaning because $4 \times 25 = 100$.

Zero in the Numerator But Not in the Denominator

$$\frac{0}{8} = ?$$

This has meaning because $0 \times 8 = 0!$

$$\frac{n}{d} = 0 \text{ if } n = 0 \text{ and } d \neq 0.$$

$$y = \frac{(x-3)}{(x+4)}$$

Application: Find the x-intercepts of:

$$y = \frac{x^2 - 4}{x^2 + 1}$$

let $y = 0$

$x = 16$

$$y = \sqrt{x}$$

$$y = \sqrt{16} = 4$$

$$(x+1) 0 = \frac{x^2 - 4}{x^2 + 1} (x^2 + 1)$$

$$0 = x^2 - 4$$

$$x^2 = 4$$

$$x = \pm 2$$

$$0 = (x - 2)(x + 2)$$

$$x = 2 \text{ OR } x = -2$$

Zero in the Denominator But Not in the Numerator

$$\frac{500}{0} = m$$

Is it possible to have $0 \times m = 500$?

$\frac{n}{d}$ is undefined if $d = 0$ and $n \neq 0$.

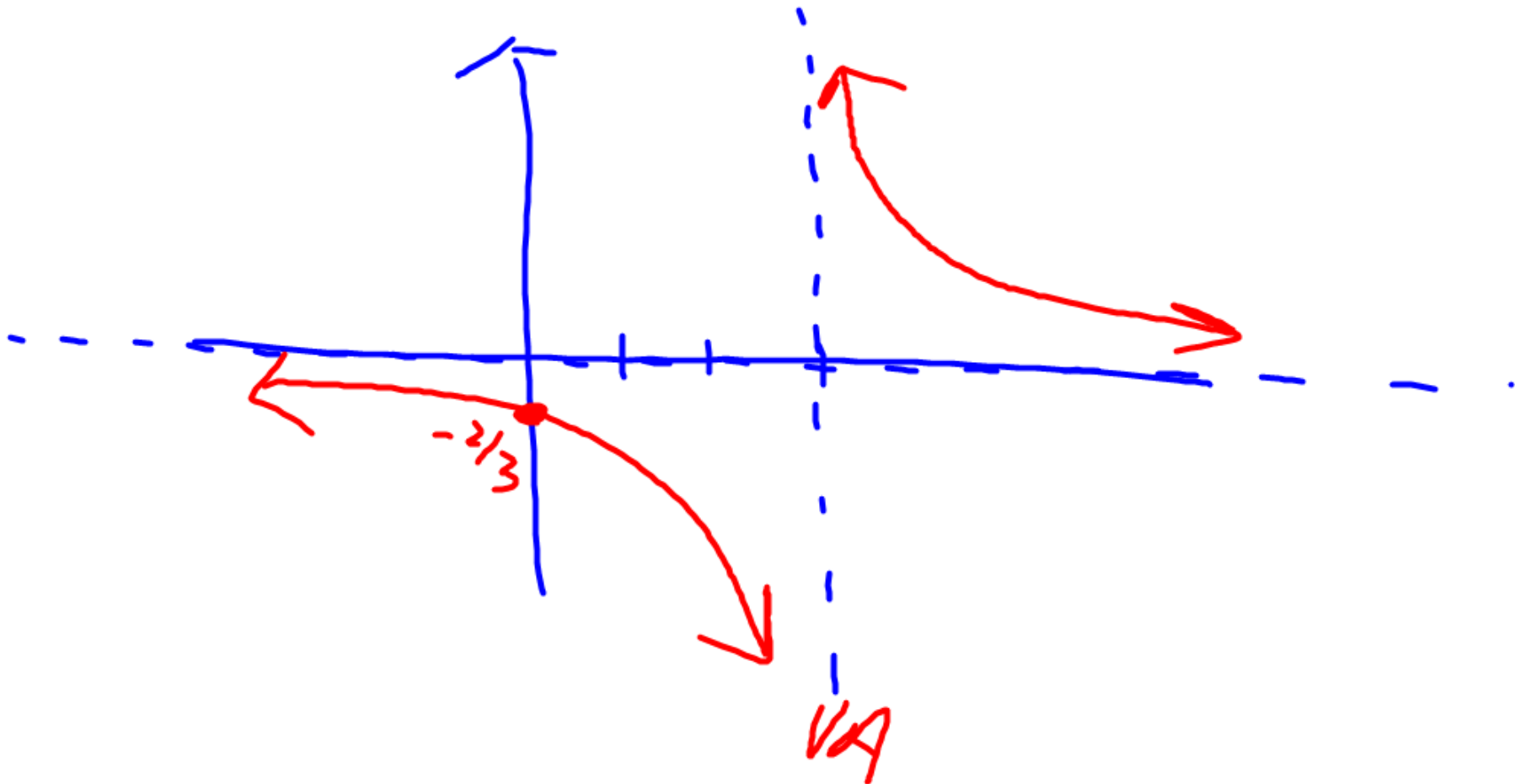


SO, I JUST DIVIDE BY ZERO AND THEN..
ZOMG!!! EVACUATE!!!!

Application: Graph the following

$$y = \frac{2}{x-3} + 0 \quad x \neq 3$$

$$y = \frac{a}{x-h} + k$$



Zero in Both the Numerator and the Denominator

$$\frac{0}{0} = k$$

Answer is not unique since $0 \times \text{anything}$ is 0 .

$\frac{0}{0}$ is indeterminate.

What are the graphical implications of this result?

Graph the following:

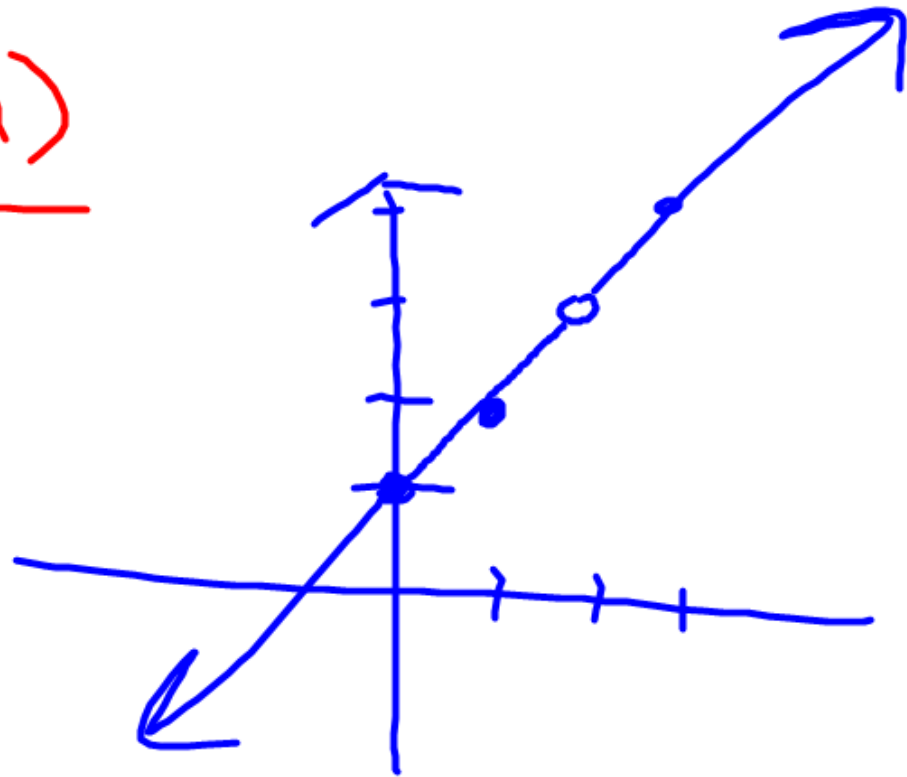
$$f(x) = \frac{x^2 - x - 2}{x - 2}$$

POD

$$x \neq 2$$

$$f(x) = \frac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}}$$

$$f(x) = x + 1$$



Graph the following:

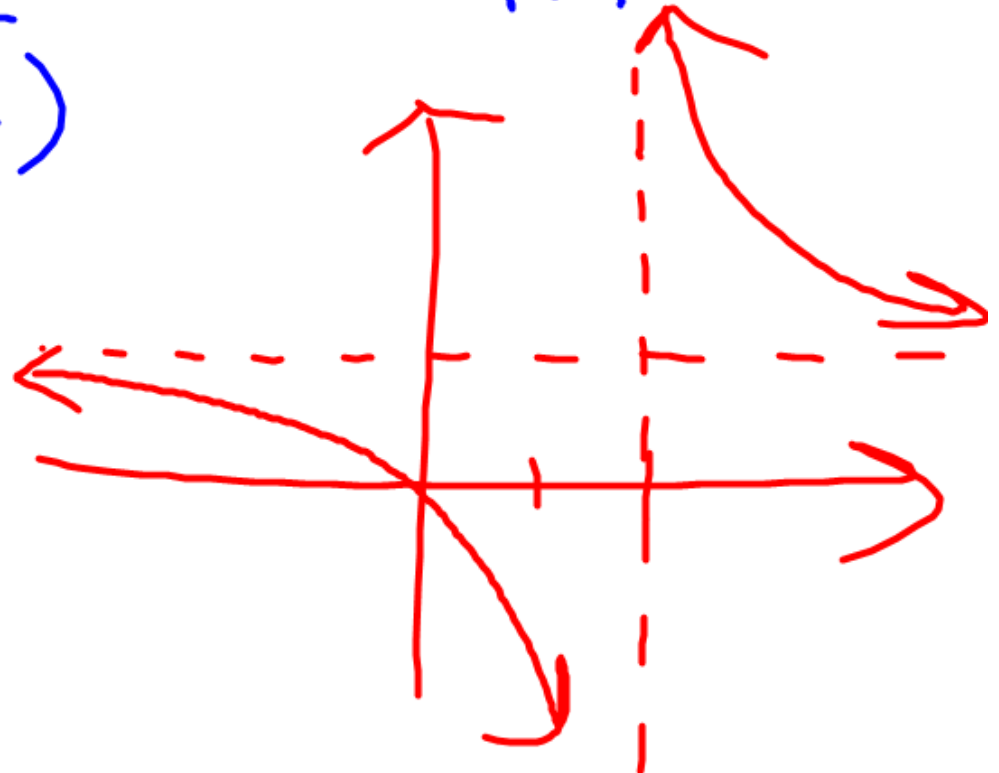
$$f(x) = \frac{x^2 - 2x}{x^2 - 4x + 4}$$

$$f(x) = \frac{x(\cancel{x-2})}{(\cancel{x-2})(x-2)}$$

$$f(x) = \frac{x}{x-2}$$

$$x \neq 2$$

VA



Here are the results of our findings!

Holes Versus Vertical Asymptote Lines

Where $f(x)$ Is Indeterminate

Suppose that $f(x) = \frac{n(x)}{d(x)} = \frac{(x-a)^p n_1(x)}{(x-a)^q d_1(x)}$ where $x-a$ is

not a factor of either $n_1(x)$ or $d_1(x)$, and p and q are positive integers. Note that $f(a)$ results in $\frac{0}{0}$, which is indeterminate.

The graphical implications follow.

- If $p \geq q$, $f(x)$ will have a hole at $x = a$.
- If $p < q$, $f(x)$ will have a vertical asymptote at $x = a$.

Ex.1 Find any values for x which the function is: a) 0 b) undefined c) indeterminate

$$f(x) = \frac{(x-2)(x+5)}{x(x-2)(x+6)}$$

$$N: \{-5, 2\}$$

$$D: \{0, 2, -6\}$$

$$a) \{-5\}$$

$$b) \{0, -6\}$$

$$c) \{2\}$$

Ex.1 Find any values for x which the function is: a) 0 b) undefined c) indeterminate

$$f(x) = \frac{x^4 - 8x}{x^2 - 4}$$

$$= \frac{x(x^3 - 8)}{(x-2)(x+2)} = \frac{x(x-2)(x^2 + 2x + 4)}{(x-2)(x+2)}$$

NO ZEROS

$$N: \{0, 2\}$$

$$D: \{-2, 2\}$$

a) $x = 0$

b) $x = -2$

c) $x = 2$

~~\emptyset~~
 $\{\emptyset\}$

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Your Turn #7

Consider the following functions. Is there a hole, a vertical asymptote line, or neither at $x = 1$?

(a) $f(x) = x - 1$

(b) $f(x) = \frac{x^2}{x-1}$

(c) $f(x) = \frac{x(x-1)}{\cancel{x-1}}$

(d) $f(x) = \frac{x\cancel{(x-1)}}{(x-1)^2}$

a) $f(1) = 1 - 1 = 0$ (1, 0)

Neither

b) $f(1) = \frac{1}{0}$ und. $x=1$ VA

c) $f(1) = \frac{0}{0}$ ind VA or **HOLE**
(1, 0)

d) $f(1) = \frac{0}{0}$ ind **VA** or HOLE

Assignment

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#'s 1-9

15, 16, 17