1.3 Division By Zero

Meaningful Division

For division to be defined and have meaning, a given quotient must have only one result that checks. Thus $\frac{100}{4} = 25$, has meaning because $4 \times 25 = 100$.

$\frac{56}{7} = 8$

Zero in the Numerator But Not in the Denominator

$$\frac{0}{8} = ?$$

This has meaning because 0x8=0!

$$\frac{n}{d} = 0 \text{ if } n = 0 \text{ and } d \neq 0.$$

Application: Find the x-intercepts of:

$$y = \frac{x^2 - 4}{x^2 + 1}$$

let $y = 0$

$(x^2+1)\ 0 = \frac{x^2-4}{x^2+1}\ (x^2+1)$

$0 = x^2 - 4$

$0 = (x-2)(x+2)$

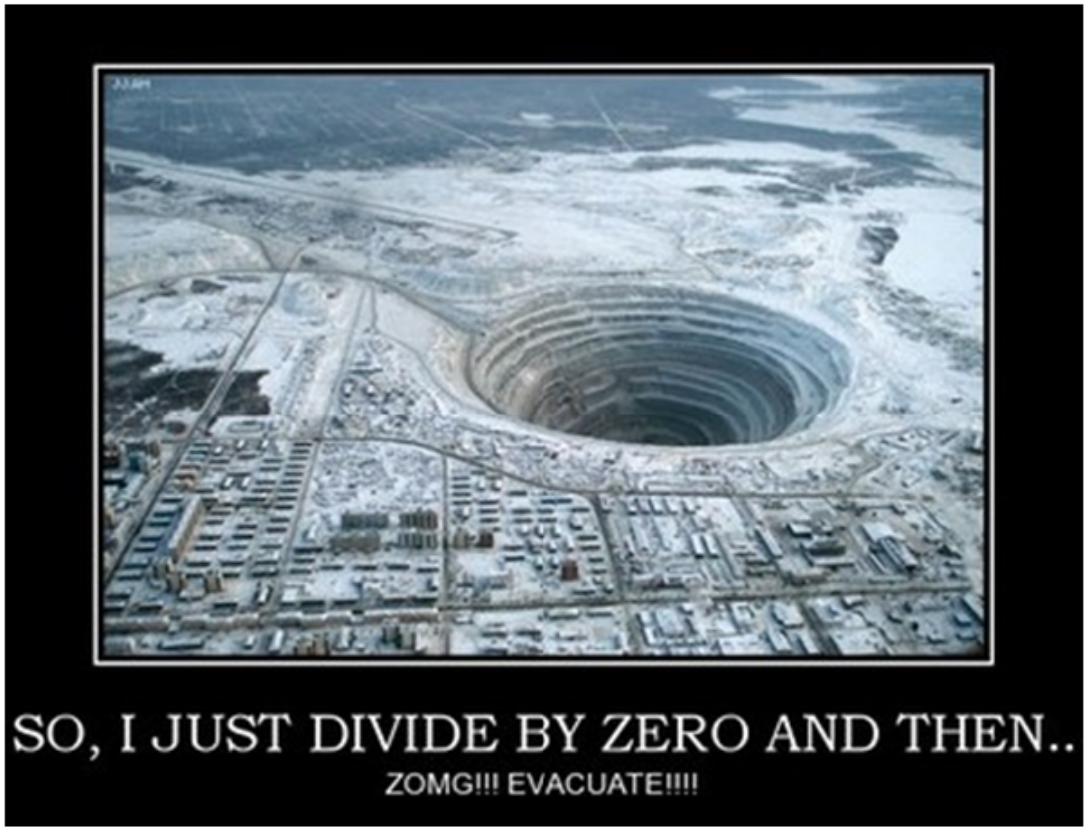
$x = 2$ or $x = -2$

Zero in the Denominator But Not in the Numerator

$$\frac{500}{0} = m$$

Is it possible to have 0xm=500?

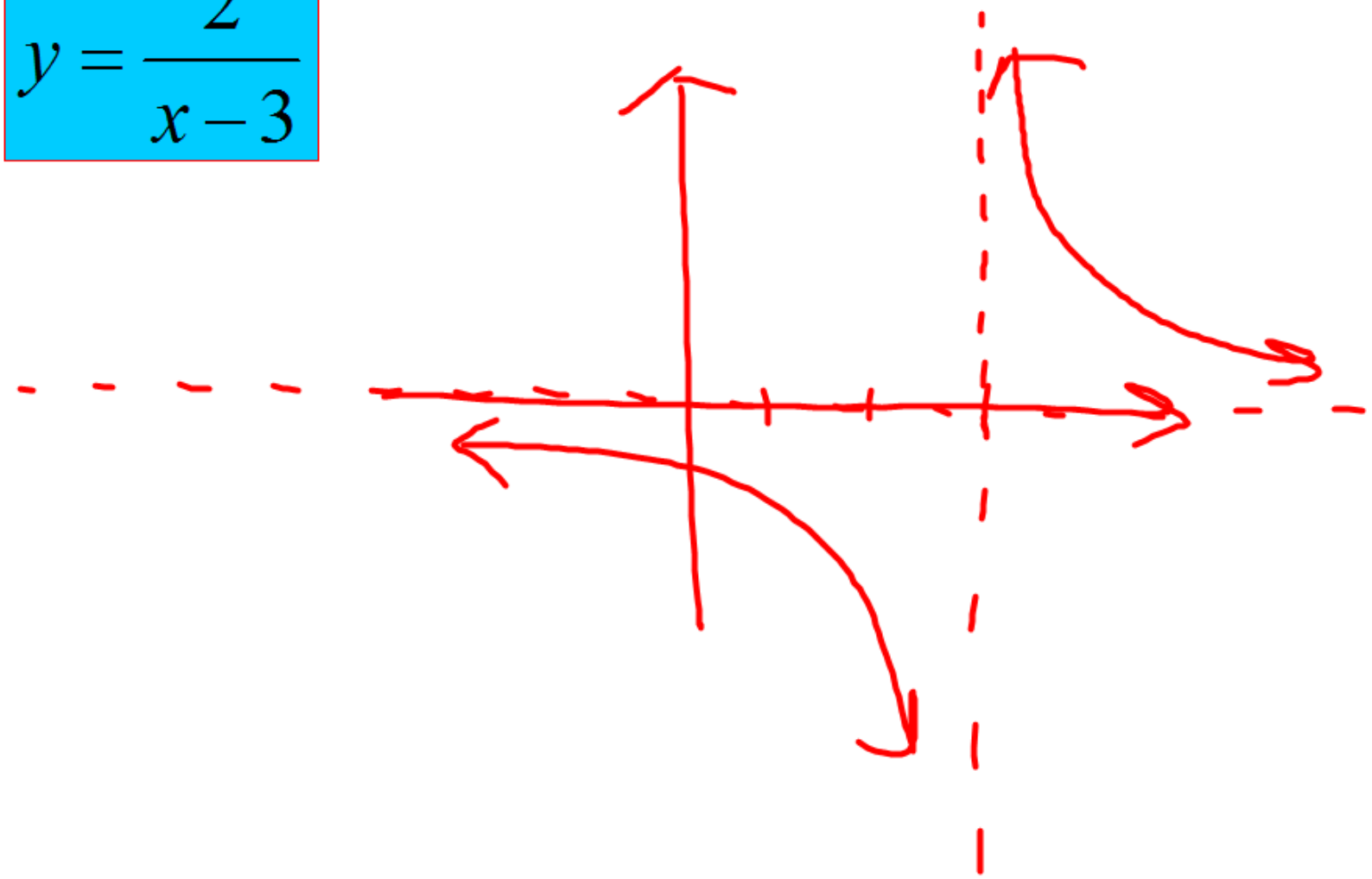
$$\frac{n}{d} \text{ is undefined if } d = 0 \text{ and } n \neq 0.$$


SO, I JUST DIVIDE BY ZERO AND THEN..
ZOMG!!! EVACUATE!!!!

Application: Graph the following

$$y = \frac{2}{x-3}$$

$x \neq 3$



Zero in Both the Numerator and the Denominator

$$\frac{0}{0} = k$$

Answer is not unique since 0 x anything is 0.

$$\frac{0}{0} \text{ is indeterminate.}$$

What are the graphical implications of this result?

Graph the following:

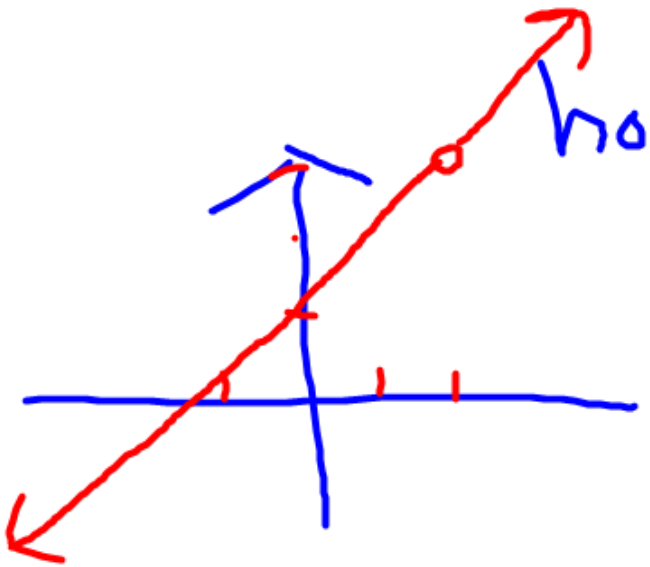
$$f(x)=\frac{x^2-x-2}{x-2}$$

$$=\frac{(x-2)(x+1)}{(x-2)}$$

$x \neq 2$

$f(x) = x+1$

hole $(2,3)$



Graph the following:

$$f(x)=\frac{x^2-2x}{x^2-4x+4} = \frac{x(x-2)}{(x-2)(x-2)}$$

$x \neq 2$

$$f(x)=\frac{x}{x-2}$$

↓

VA

Here are the results of our findings!

Holes Versus Vertical Asymptote Lines Where $f(x)$ Is Indeterminate

Suppose that $f(x)=\dfrac{n(x)}{d(x)}=\dfrac{(x-a)^p\, n_1(x)}{(x-a)^q\, d_1(x)}$ where $x-a$ is not a factor of either $n_1(x)$ or $d_1(x)$, and p and q are positive integers. Note that $f(a)$ results in $\dfrac{0}{0}$, which is indeterminate.

The graphical implications follow.

- If $p \geq q$, $f(x)$ will have a hole at $x=a$.
- If $p<q$, $f(x)$ will have a vertical asymptote at $x=a$.

Ex.1 Find any values for x which the function is: a) 0 b) undefined c) indeterminate

$$f(x)=\frac{(x-2)(x+5)}{x(x-2)(x+6)}$$

$N=\{-5,2\}$

$D=\{-6,0,2\}$

a) $\{-5\}$

b) $\{-6,0\}$

c) $\{2\}$

Ex.1 Find any values for x which the function is: a) 0 b) undefined c) indeterminate

$$f(x)=\frac{x^4-8x}{x^2-4}=\frac{x(x-2)(x^2+2x+4)}{(x-2)(x+2)}$$

$N=\{0,2\}$

$D=\{-2,2\}$

a) $\{0\}$

b) $\{-2\}$

c) $\{2\}$

Your Turn #7

Consider the following functions. Is there a hole, a vertical asymptote line, or neither at $x = 1$?

(a) $f(x) = x - 1$

(b) $f(x) = \dfrac{x^2}{x-1}$

(c) $f(x) = \dfrac{x(x-1)^3}{x-1}$

(d) $f(x) = \dfrac{x(x-1)}{(x-1)^2}$

a) $f(1) = (1-1) = 0$

point $(1,0)$

b) $f(1) = \frac{1}{0}$ $x=1 \rightarrow$ VA

c) $f(1) = \frac{0}{0}$ hole $x=1$

d) $f(1) = \frac{0}{0}$ VA $x=1$

Assignment
Page 24
#'s1-9, 15, 16, 17

⑦ $f(x) = \dfrac{x^2-9}{x^2-5x+6} = \dfrac{(x-3)(x+3)}{(x-3)(x-2)}$

N: $\{3, -3\}$

D: $\{3, 2\}$

i) $\{-3\}$

ii) $\{2\}$

iii) $\{3\}$

15

$$f(x) = \frac{27x - 3^x}{3^x - 9x} = \frac{x(27 - 3^x)}{x(3^x - 9)}$$

N: $\{0, 3\}$

D: $\{0, 2\}$

i) $\{3\}$

ii) $\{2\}$

iii) $\{0\}$

15

$$f(x) = \frac{27x - 3^x}{3^x - 9x} = \frac{x(27 - 3^x)}{x(3^x - 9)}$$

N: $\{0, 3\}$

D: $\{0, 2\}$

i) $\{3\}$

ii) $\{2\}$

iii) $\{0\}$

$x = 4$

a) $f(x) = (x-4)^2$

$x=4$ defined

b) $f(x) = \dfrac{x+4}{(x-4)^2} = \dfrac{8}{0}$

$x=4$ VA

c) $f(x) = \dfrac{(x-4)^3}{x-4}$

IND → HOLE

↘ VA

$f(x) = x-4$

HOLE

$(4, 0)$

$x=4$

d) $f(x)=\dfrac{x-4}{x+4}$

defined

$(4, 0$

e) $f(x)=\dfrac{x^3(x-4)^2}{(x-4)^3}$

$f(4)=\dfrac{0}{0}$ IND → Hole, → VA

f) $f(x)=\dfrac{10(x-4)^5}{(x-4)^5}$ $f(x)=\dfrac{x^3}{x-4}$

$f(4)=\dfrac{0}{0}$ IND → H, → VA

$\frac{0}{\#}$ x value x int

$\frac{\#}{0}$ undefined VA

$\frac{0}{0}$ indertimate