1.3 Combining Transformations

Multiple transformations can be applied to a function using the general transformation model :

$y = af(b(x\text{-}h)) + k$ or $y\text{-}k = af(b(x\text{-}h))$

$$y = af(b(x-h)) + k$$

$a \rightarrow$ vert. stretch by a factor of a, and if $a < 0$ reflection x axis.

$b \rightarrow$ horizontal stretch by a factor of $\frac{1}{b}$, and if $b < 0$ reflection y axis.

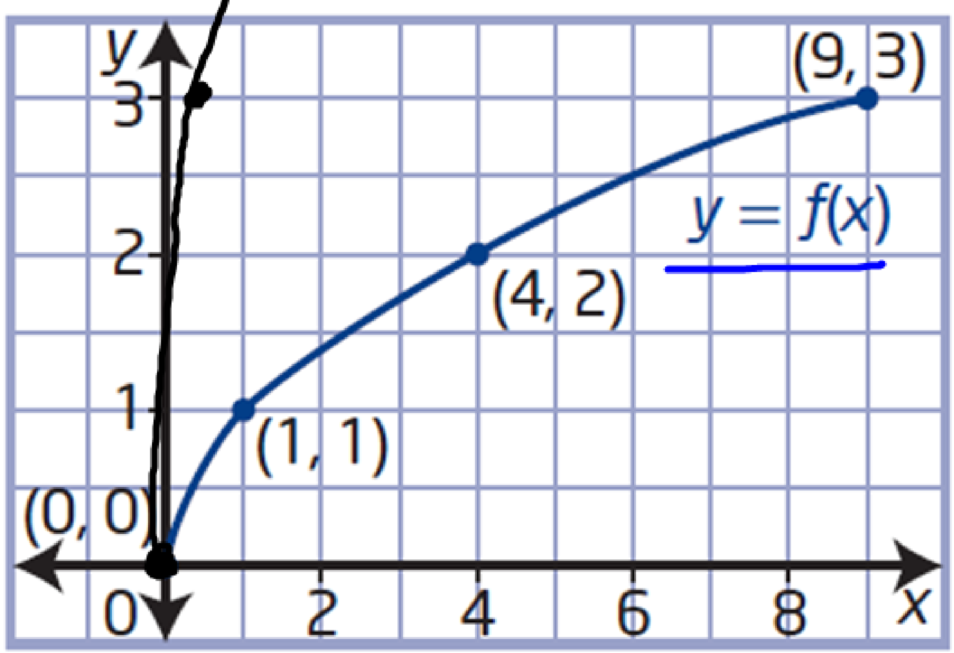
$h \rightarrow$ horizontal shift.

$k \rightarrow$ vertical shift.

$(x, y) \longrightarrow \left(\frac{x}{b} + h,\ ay + k\right)$

Example 1: Describe the combination of transformation s that must be applied to the function *y=f(x)* to obtain the transformed function. Sketch the transformed function.

a) $y = 3f(2x)$



$(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$

$a = 3$ vert. stretch by factor of 3.

$b = 2$ horizontal stretch by a factor of $\frac{1}{2}$

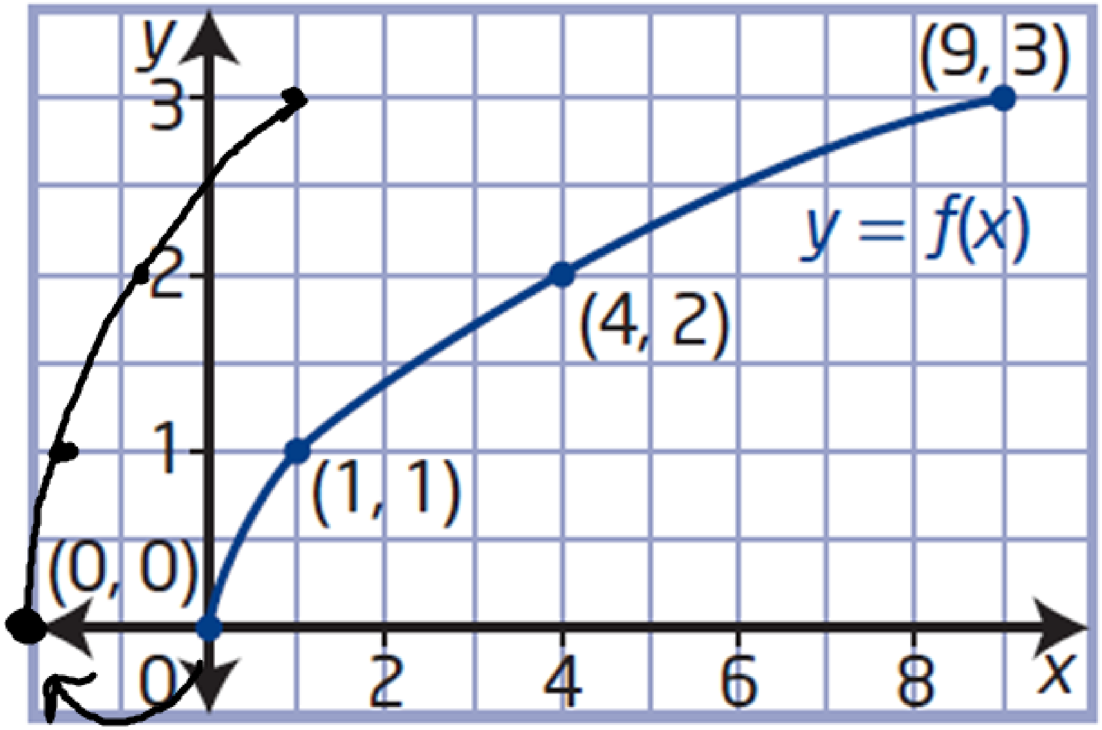
x	y
0	0
1	1
4	2
9	3

$\left(\frac{x}{2}, 3y\right)$

x	y
0	0
$\frac{1}{2}$	3
2	6
$\frac{9}{2}$	9

$y = f(3(x+2))$

b) $y = f(3x+6)$



$(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$

$b = 3$ horizontal stretch by factor $\frac{1}{3}$

$h = -2$ shift 2 units left.

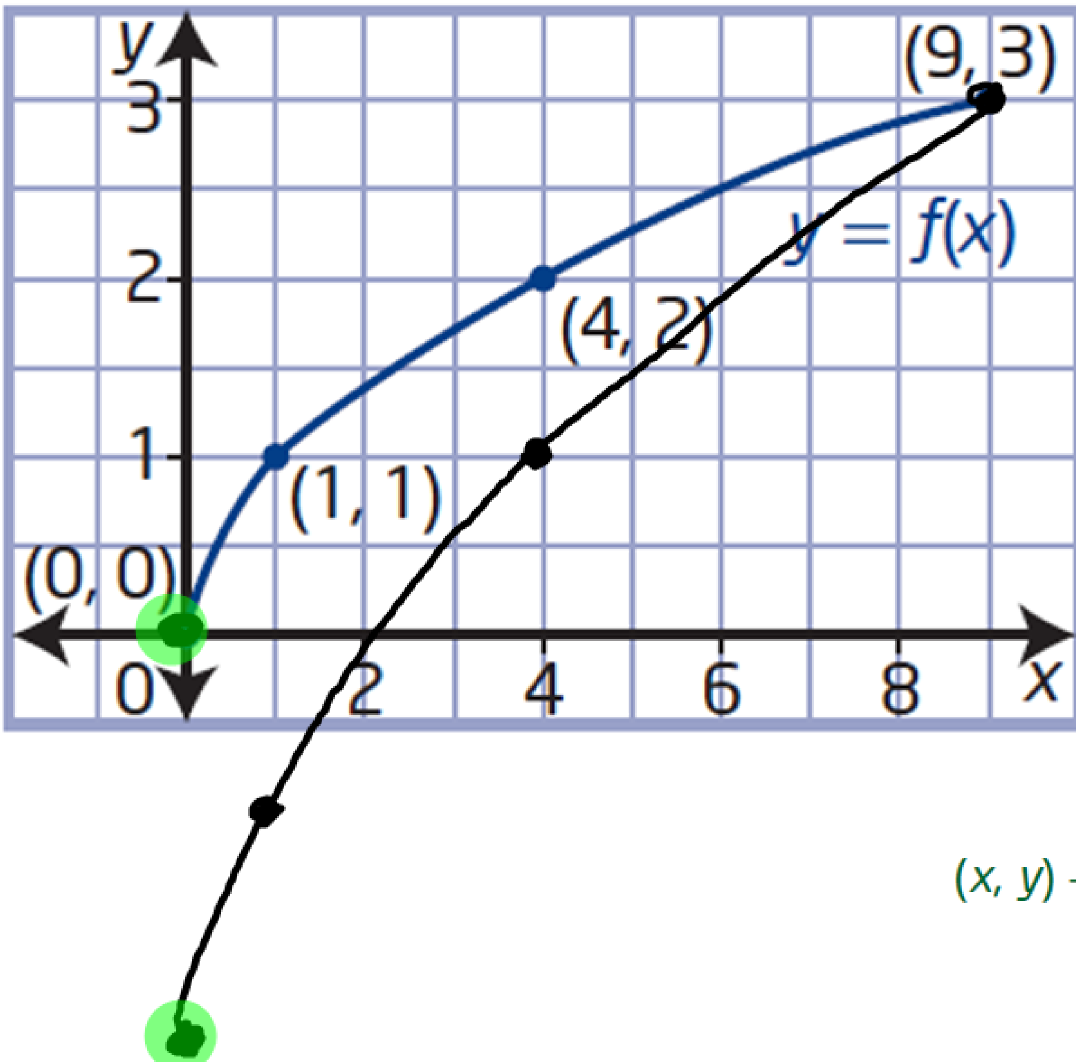
$$(x, y) \longrightarrow \left(\frac{x}{3} - \frac{6}{3}, \; y\right)$$

x	y
0	0
1	1
4	2
9	3

x	y
-2	0
$-5/3$	1
$-2/3$	2
1	3

Your Turn

c) $y = 2f(x) - 3$



$(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$

$a = 2$ vert. stretch by factor of 2.

$k = -3$ shift 3 units down

$(x, 2y - 3)$

x	y
0	0
1	1
4	2
9	3

x	y
0	-3
1	-1
4	1
9	3

Combination of Transformations

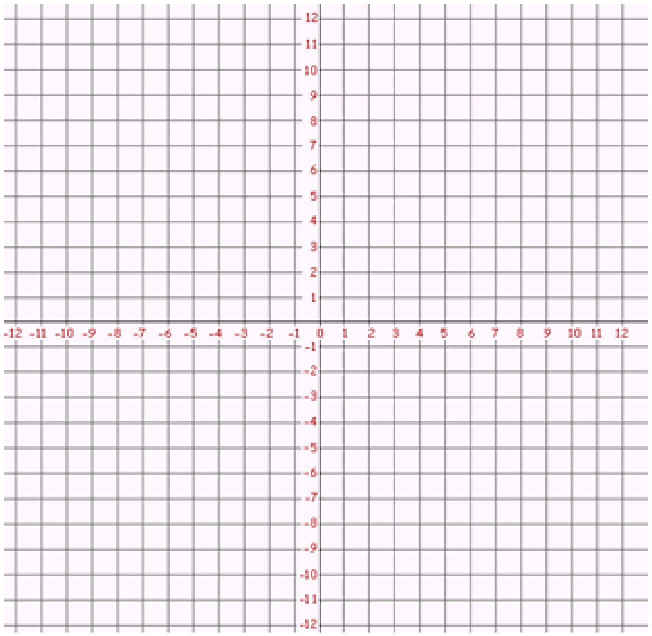
Show the combination of transformations that should be applied to the graph of the function $f(x) = x^2$ in order to obtain the graph of the transformed function $g(x) = -\frac{1}{2}f(2(x - 4)) + 1$. Write the corresponding equation for $g(x)$.

For $g(x) = -\frac{1}{2}f(2(x - 4)) + 1$, $a = -\frac{1}{2}$, $b = 2$, $h = 4$, and $k = 1$.

The equation of the transformed function is $g(x) = -\frac{1}{2}(2(x-4))^2 + 1$.

Your Turn

Describe the combination of transformations that should be applied to the function $f(x) = x^2$ in order to obtain the transformed function $g(x) = -2f\left(\frac{1}{2}(x + 8)\right) - 3$. Write the corresponding equation and sketch the graph of $g(x)$.



$$(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$$

$a=-2$ reflection x axis, vert. st. by factor 2.

$b=\frac{1}{2}$ h. stretch by factor of 2

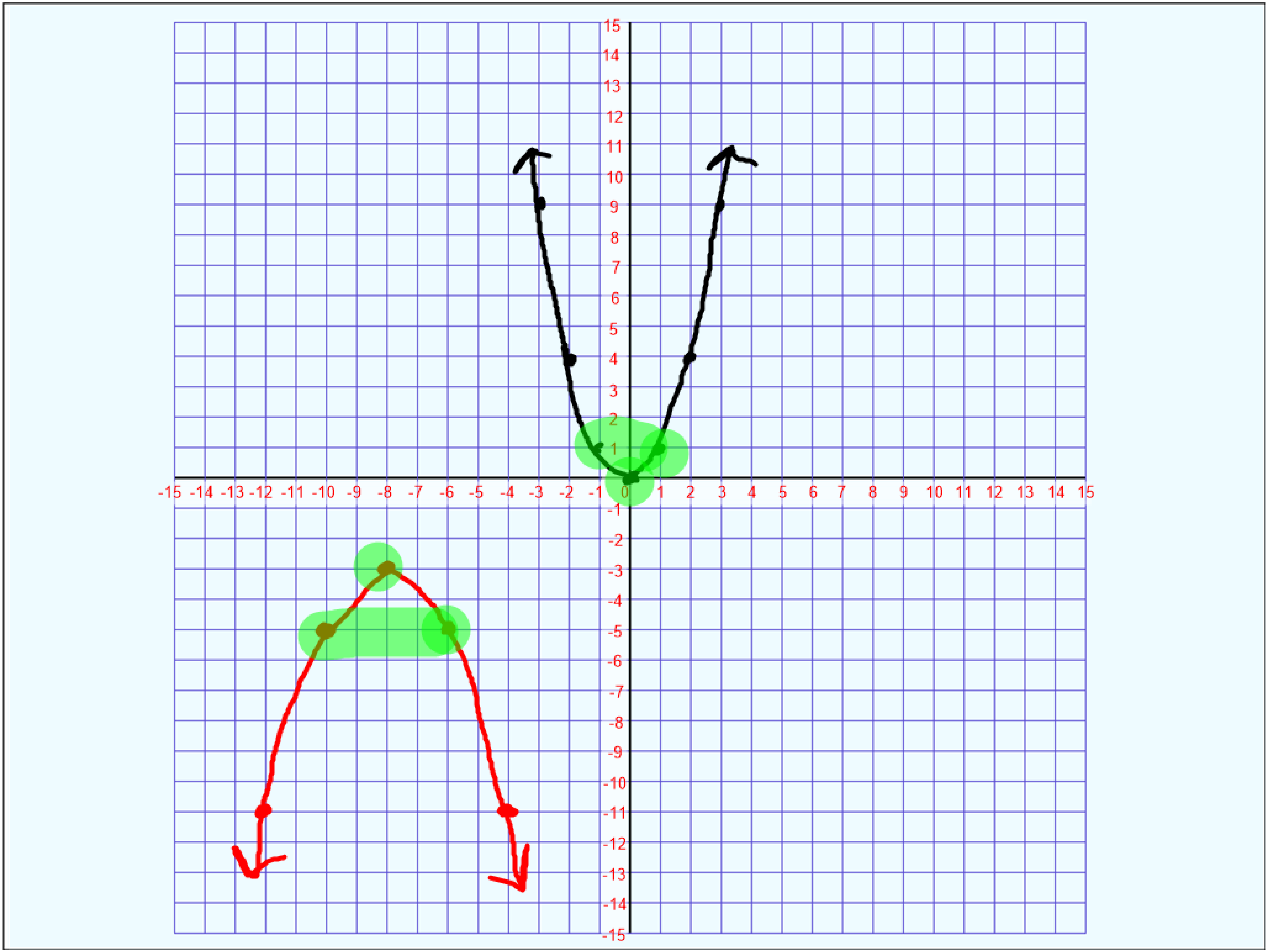
$h=-8$ shift 8 left

$k=-3$ shift 3 down

$$(x, y) \rightarrow (2x - 8, -2y - 3)$$

x	y
0	0
1	1
-1	1
2	4
-2	4

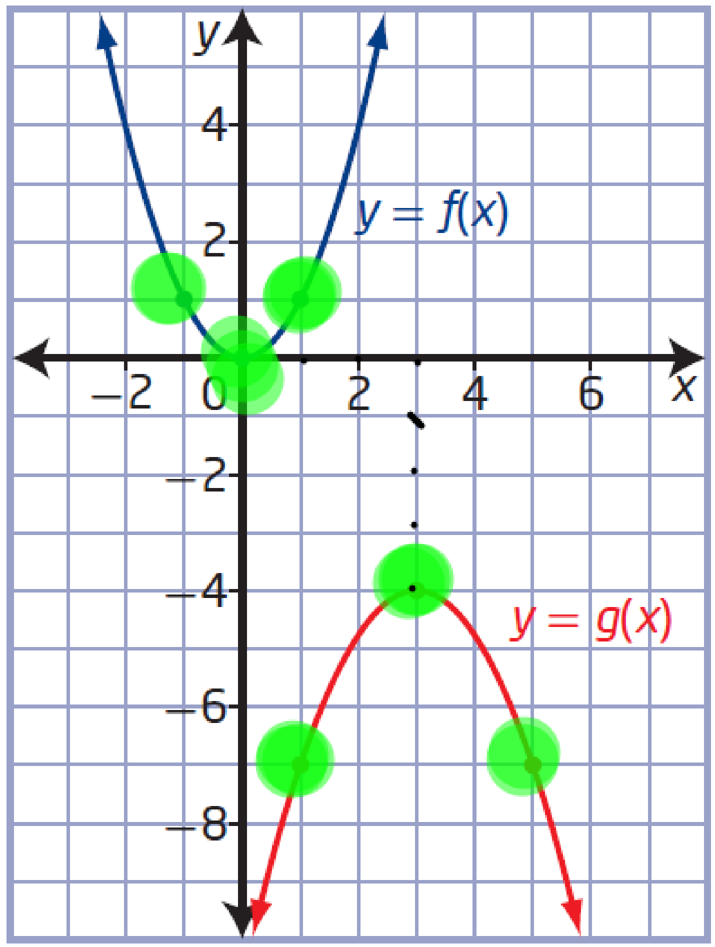
x	y
-8	-3
-6	-5
-10	-5
-4	-11
-12	-11


-15 -14 -13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 -1 -2 -3 -4 -5 -6 -7 -8 -9 -10 -11 -12 -13 -14 -15

$g(x) = -3f\left(\frac{1}{2}(x-3)\right) + 4$

Your turn!

The graph of the function $y = g(x)$ represents a transformation of the graph $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x-h))+k$. Explain your answer.



$a = -3$

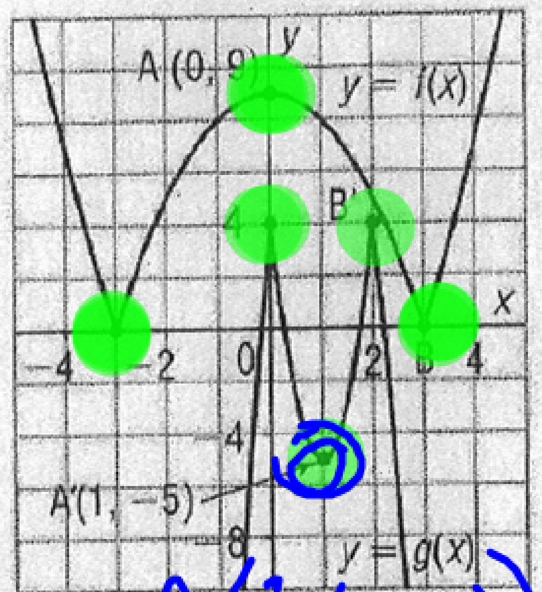
$b = \frac{1}{2}$

$h = 3$

$k = 4$

The graph of $y = g(x)$ is the image of the graph of $y = f(x)$ after a combination of transformations. Corresponding points are labelled. Write an equation of each image graph in terms of the function f.

a)



write your answer in the form $y = a f(b(x-h)) + k$.

$a = -1$

$b = 3$

$h = 1$

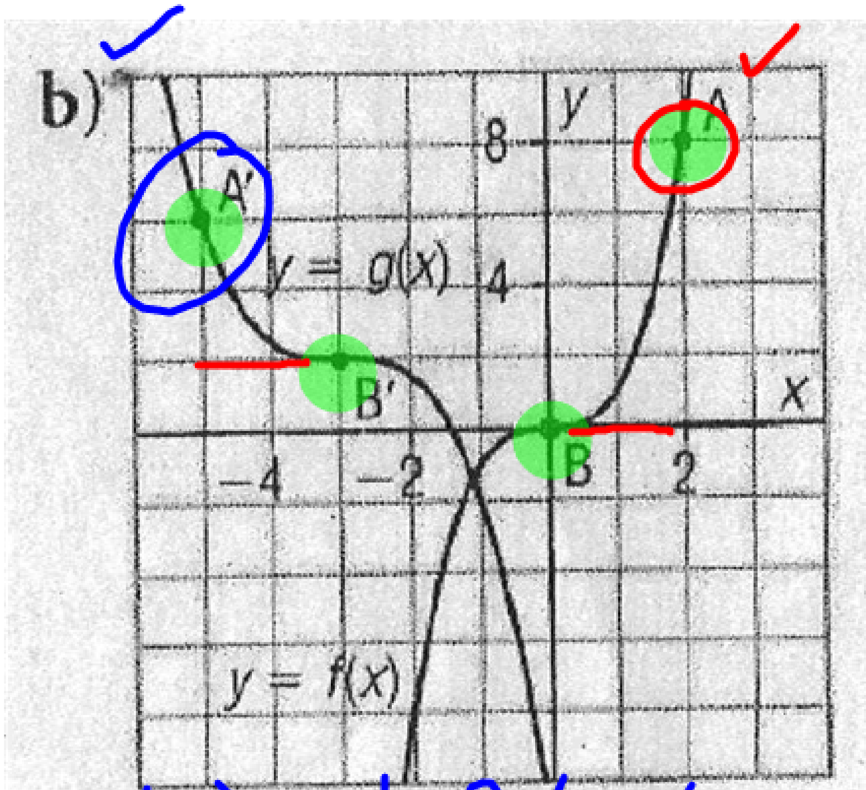
$k = 4$

$g(x) = -f(3(x-1)) + 4$

$(0, 9) \rightarrow \left(\frac{x}{3} + h, -y + k\right) \rightarrow (1, -5)$

$\frac{0}{3} + h = 1 \qquad -9 + k = -5$

$k = 4$



$a = \frac{1}{2}$

$b = -1$

$h = -3$

$k = 2$

$g(x) = \frac{1}{2} f(-(x+3)) + 2$

$(2, 8) \longrightarrow \left(-x+h, \frac{1}{2}y+k\right) \rightarrow (-5, 6)$

$-2 + h = -5$

$h = -3$

$\frac{1}{2}(8) + k = 6$

$4 + k = 6$

$k = 2$

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#'s 1,2,4,5,6,7,8,9a,b,c,e,10

11,12,13

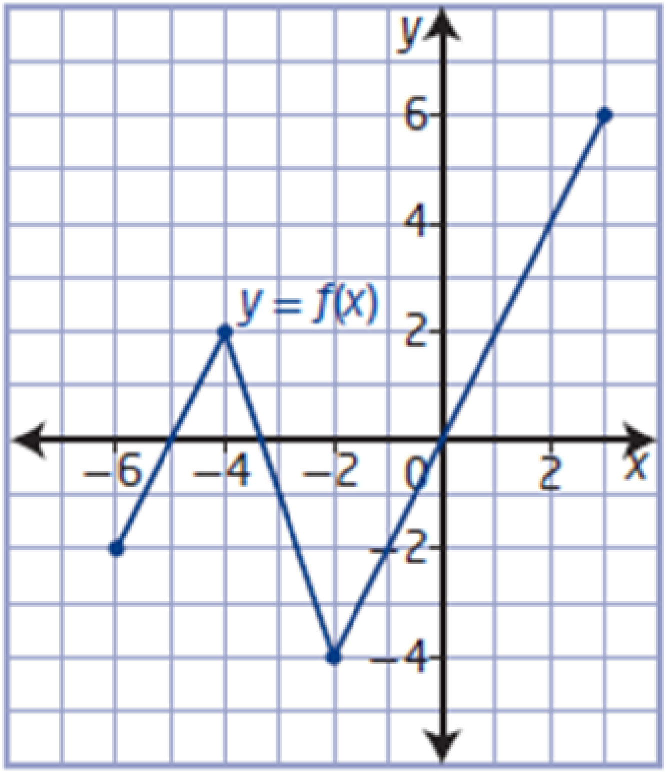
a

b

h

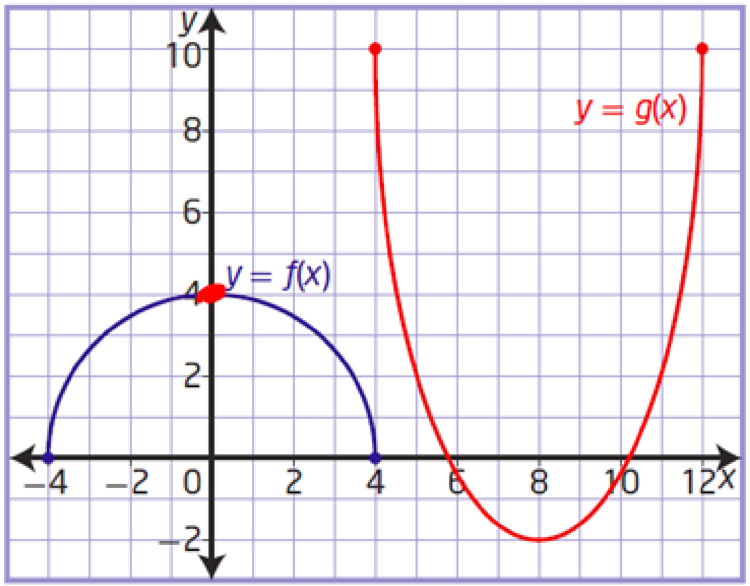
k

9. Given the graph of $y = f(x)$, sketch the graph of $y - 4 = -\frac{1}{4}f\left(\frac{1}{2}(x+3)\right)$.

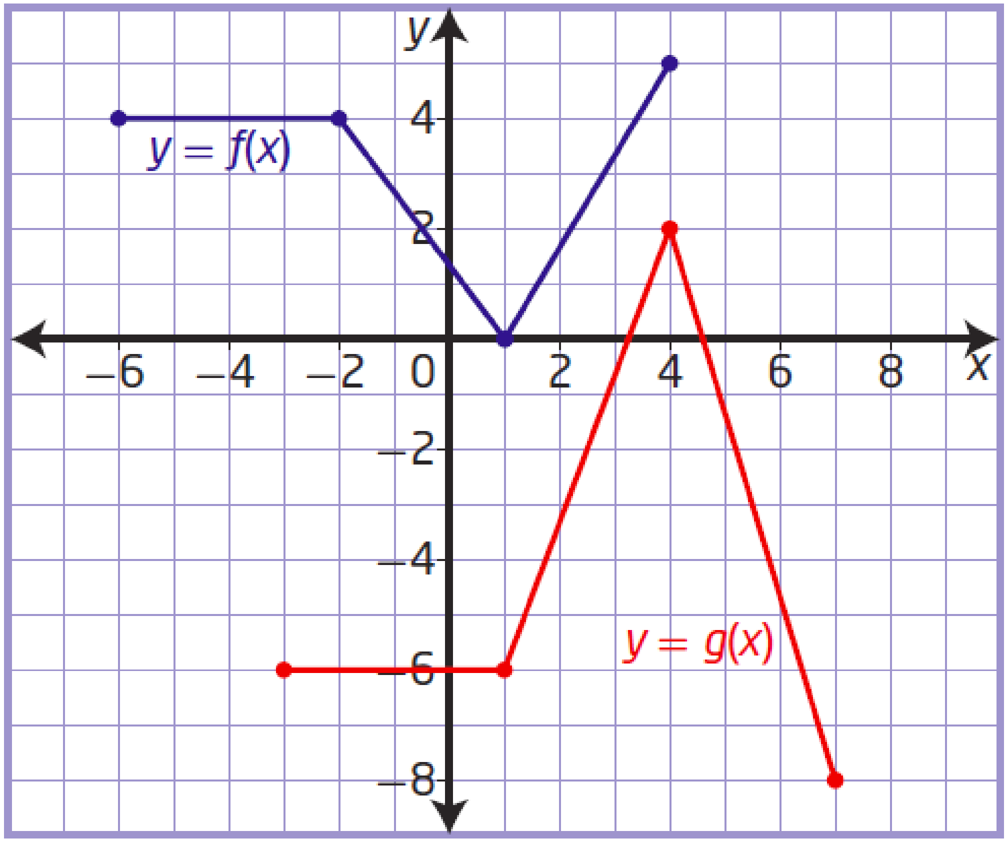


10. The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.

a)



b)



c)