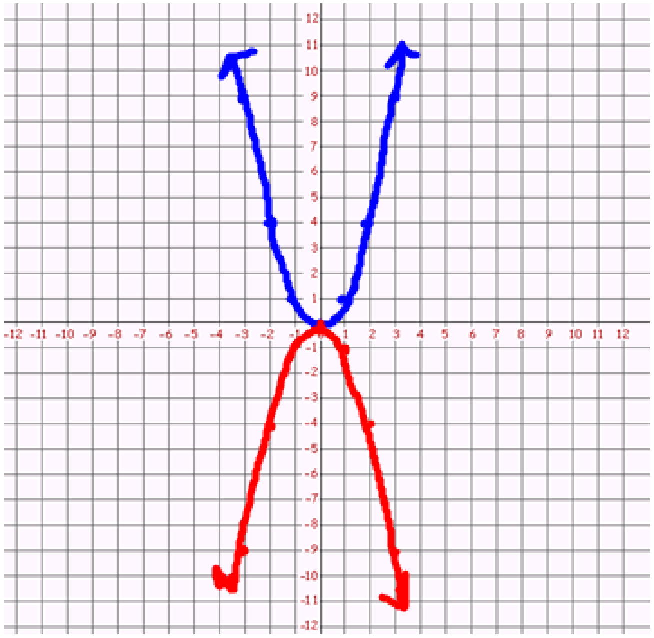
$y - k = f(x - h)$

1.2 Reflections and Stretches

$y = af(bx)$

Let's examine the function $y = x^2$.

Let's compare to the function $y = -x^2$.

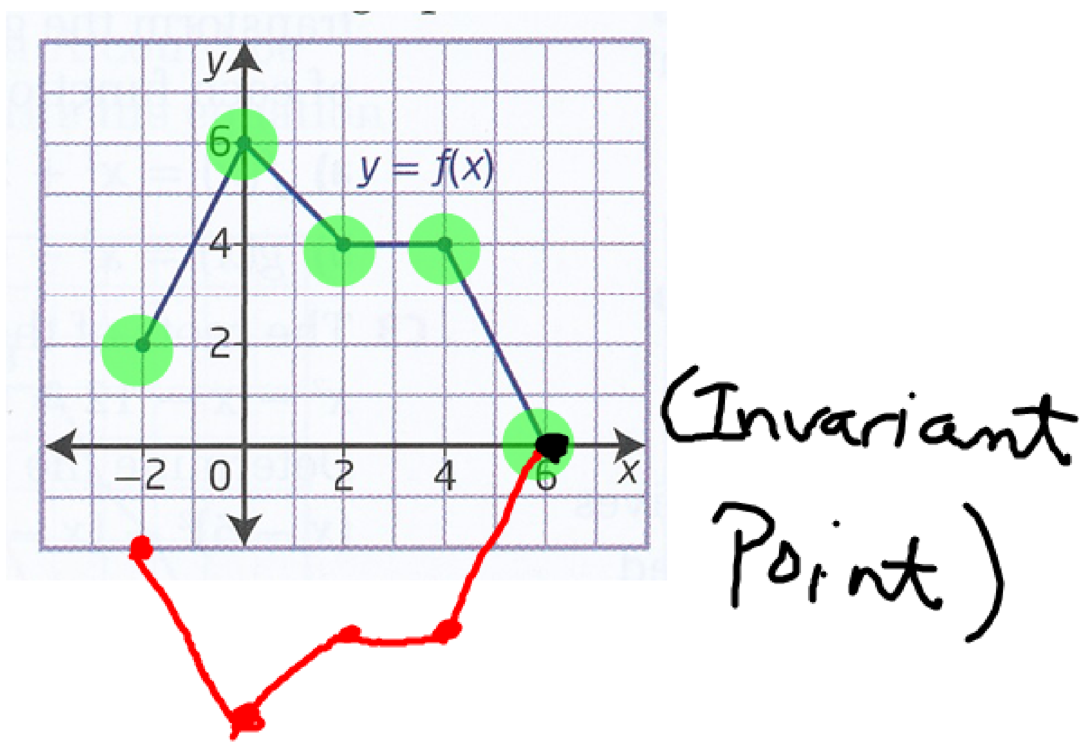


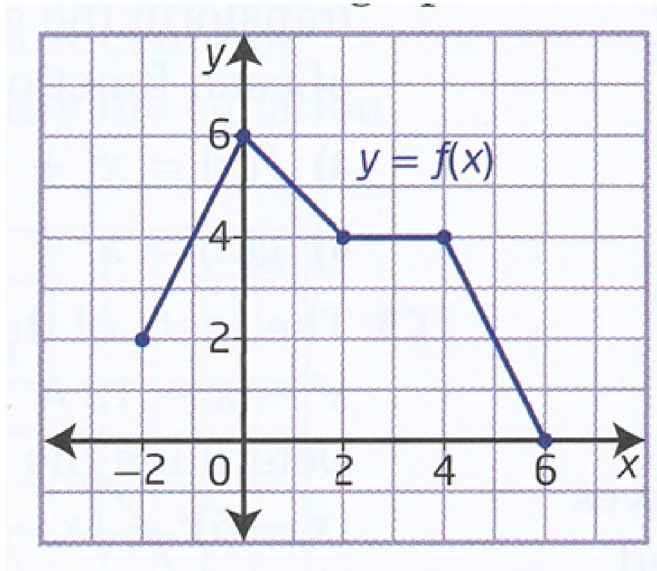
Reflection About The x-Axis

A **reflection** of a graph creates a mirror image in a line called the line of reflection. Reflections, like translations, do not change the shape of a graph, they only change the **orientation**.

When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x - axis.

Given $y=f(x)$, sketch $y=-f(x)$



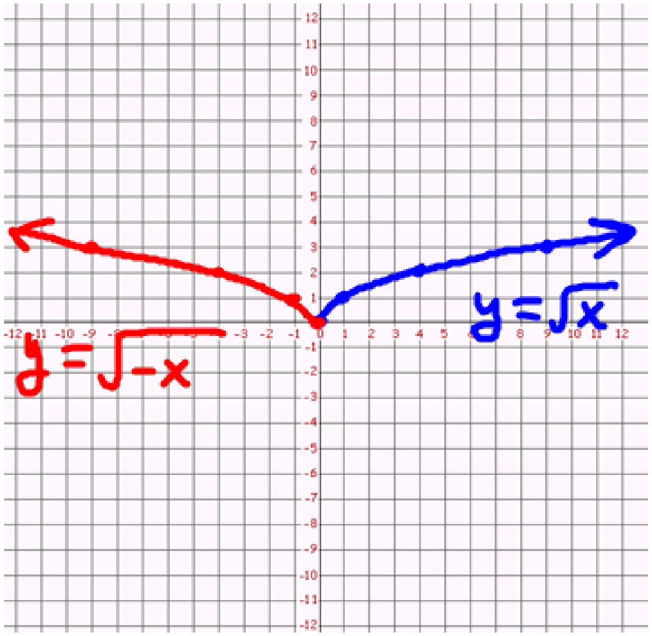


Notice the point (6,0) maps itself C’(6,0). This point is called an **invariant point.**

An **invariant point** is a point on the graph that remains unchanged after a transformation is applied to it. Any point on a curve that lies on the line of reflection is an invariant point.

Let's examine the function $y = \sqrt{x}$.

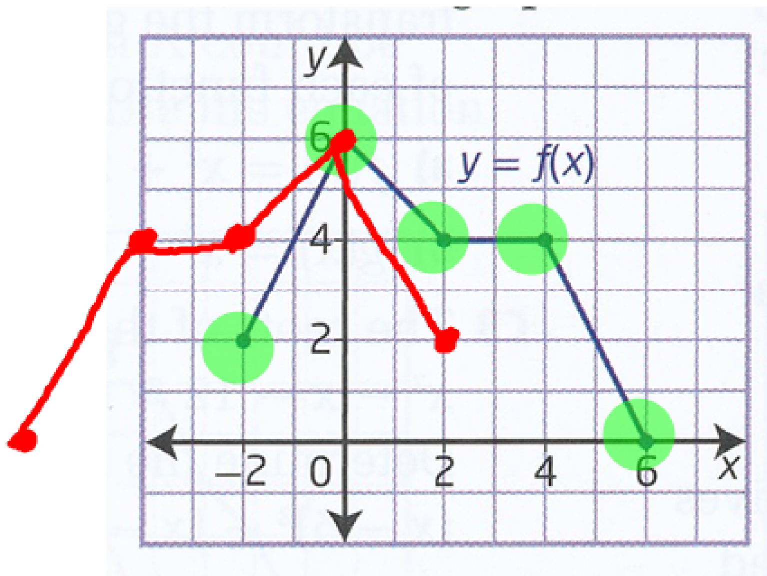
Let's examine the function $y = \sqrt{-x}$



Reflection About The y-Axis

When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

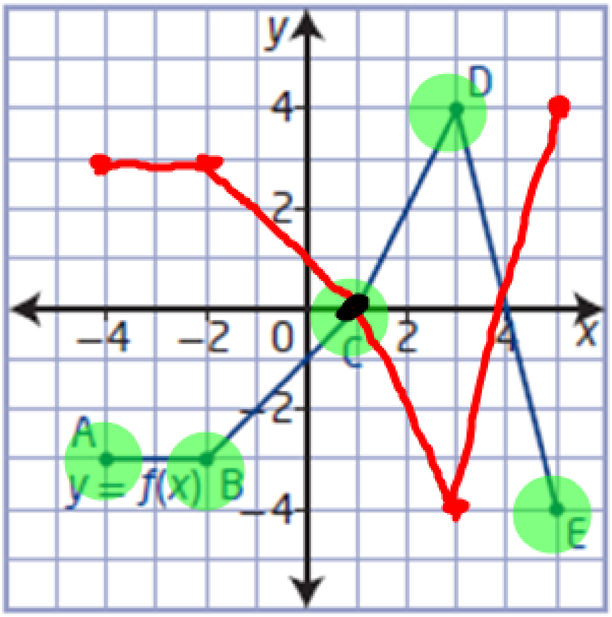
Given $y=f(x)$, sketch $y=f(-x)$



So to review:

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.
- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

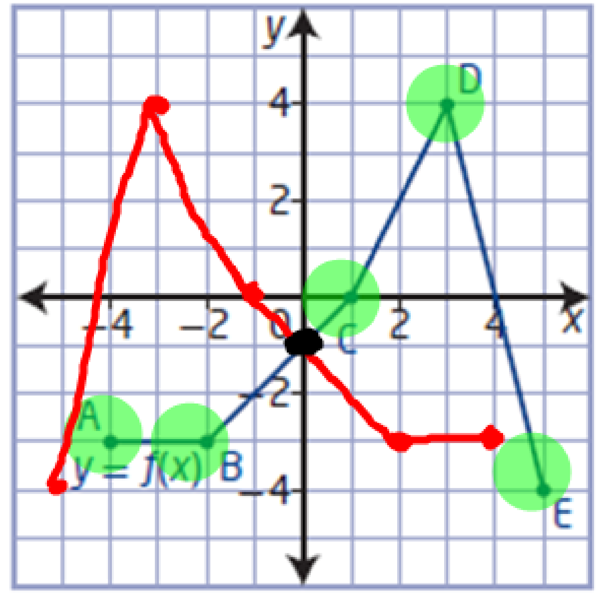
Given y=f(x), sketch y=-f(x)



(1, 0)

What are the invariant points?

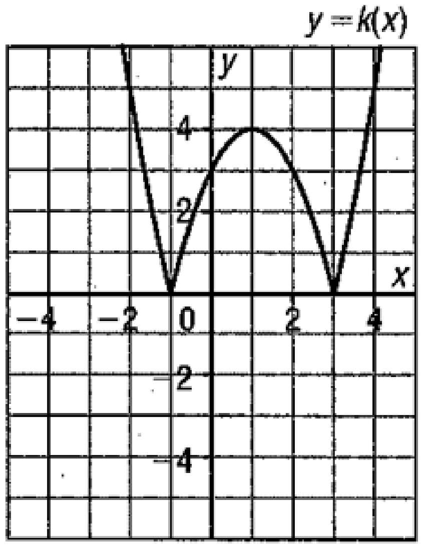
Given *y=f(x)*, sketch *y=f(-x)*



(0, -1)

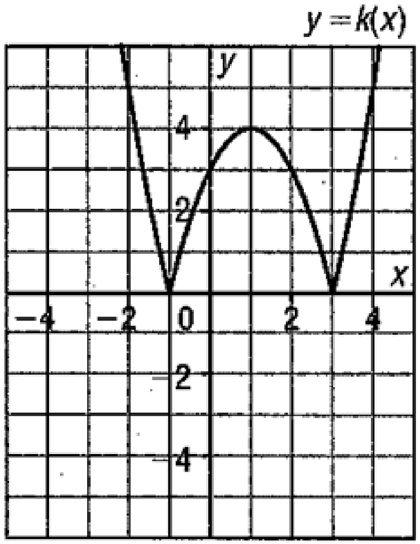
Here is the graph of $y = k(x)$. On the same grid, sketch and label the graph of each function below. State the domain and range of each function.

a) $y = -k(x)$



What are the invariant points?

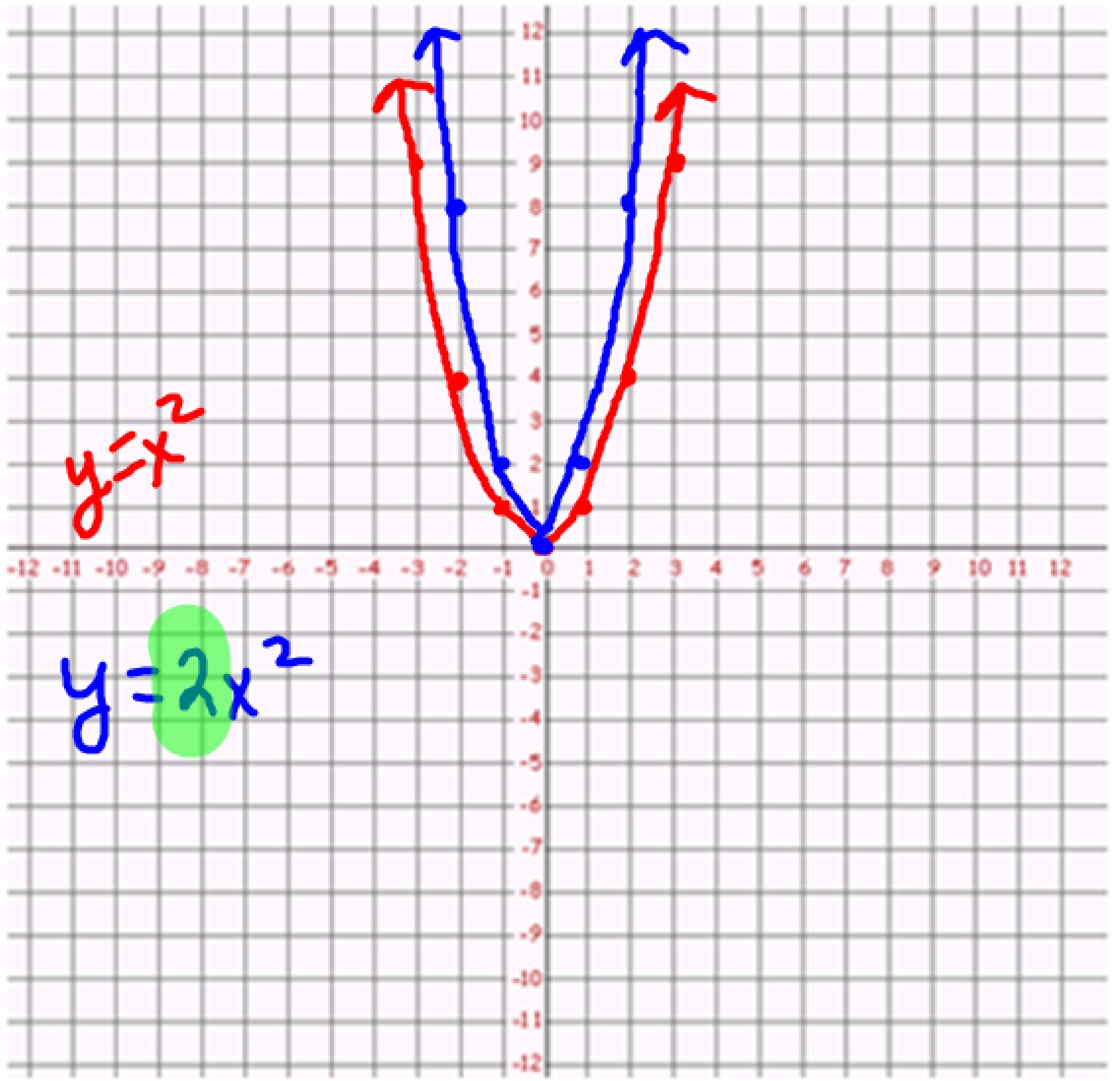
b) $y = k(-x)$



Vertical Stretch

Graph $y = x^2$

Compare to $y = 2x^2$



A **stretch**, unlike a translation or reflection, changes the shape of the graph. However like translations, stretches do not change the orientation of the graph.

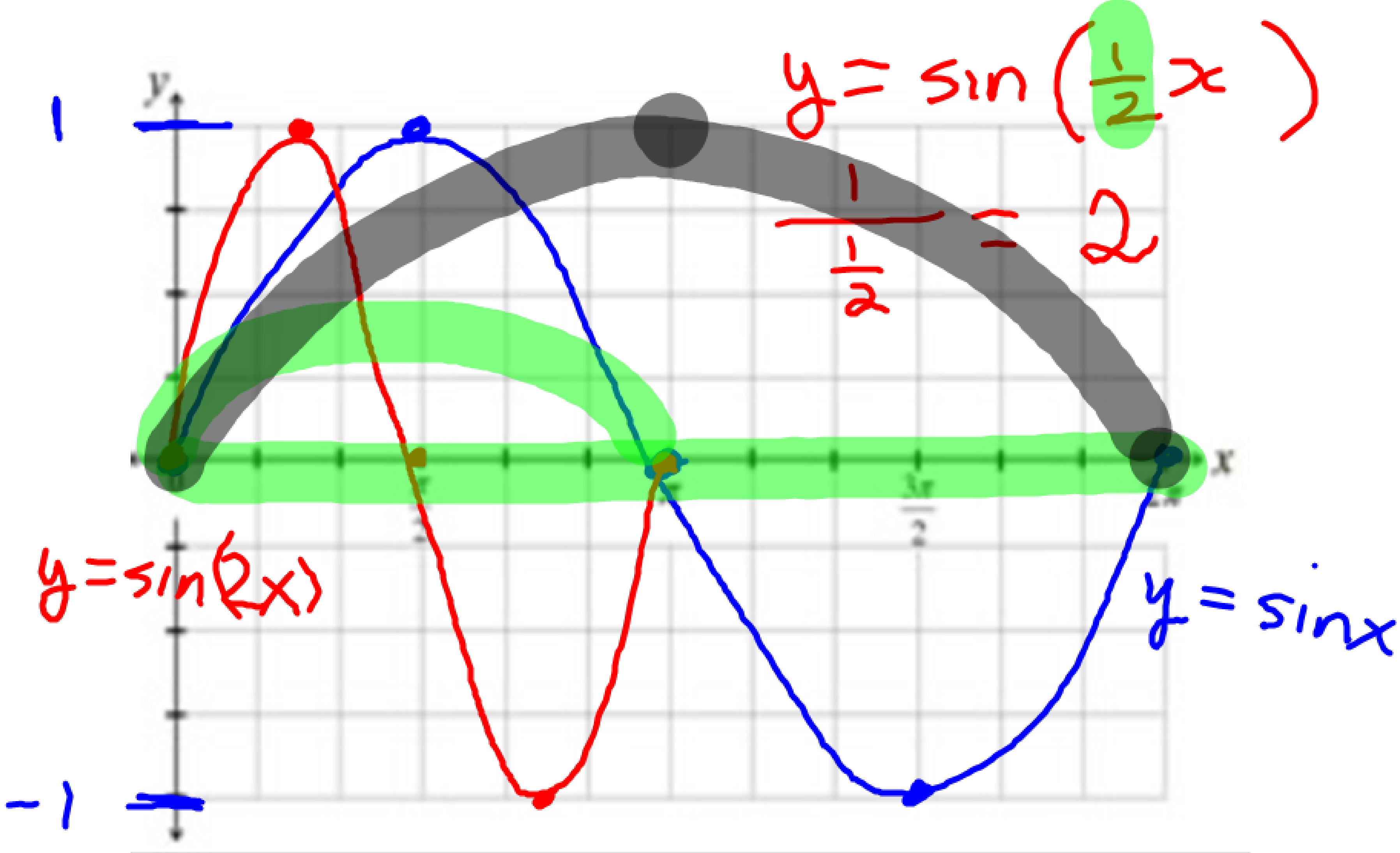
Vertical Stretch

When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.

Horizontal Stretches

y=sin x

y=sin(2x)



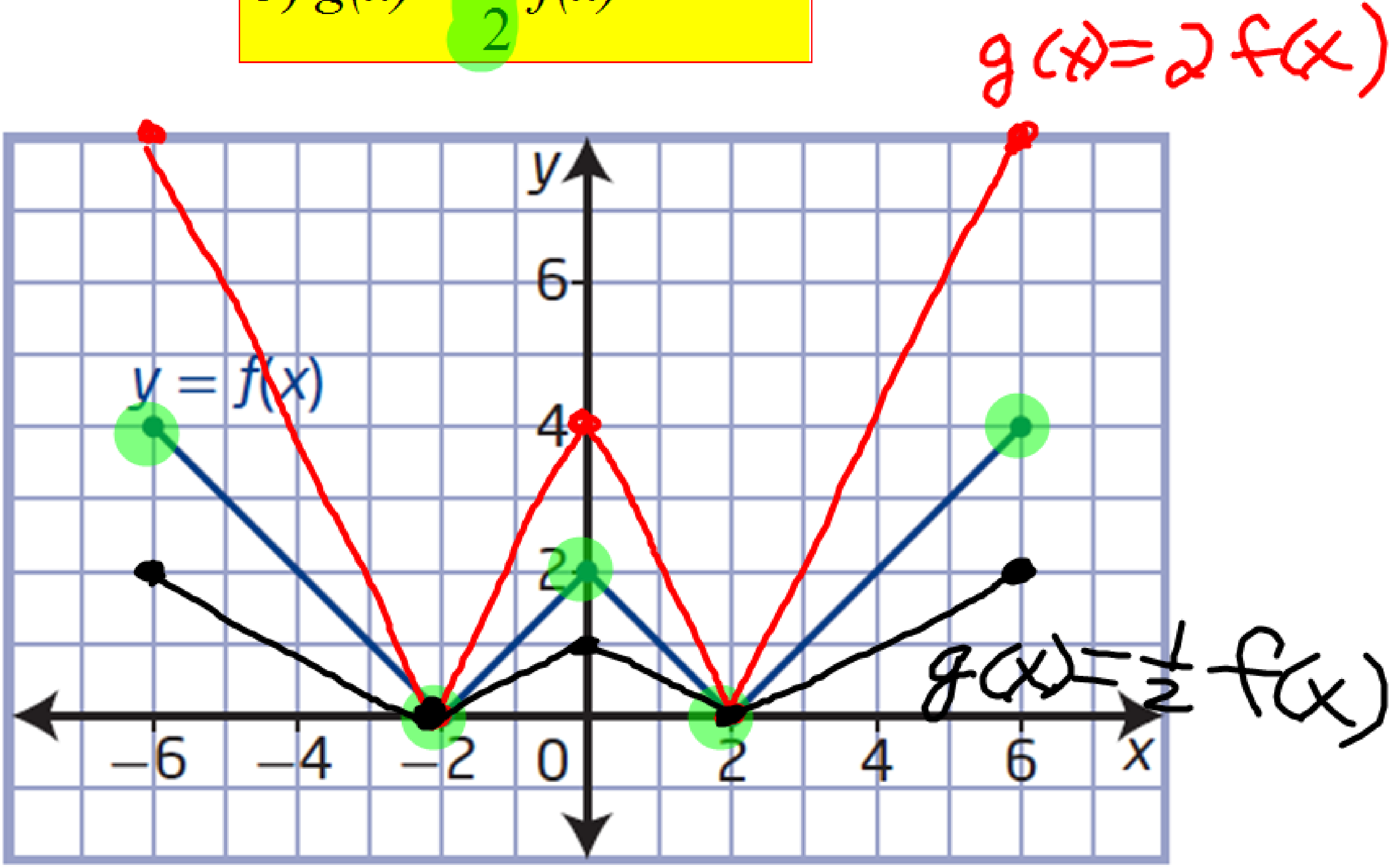
Horizontal Stretches

When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

Given $y = f(x)$, graph :

a) $g(x) = 2f(x)$

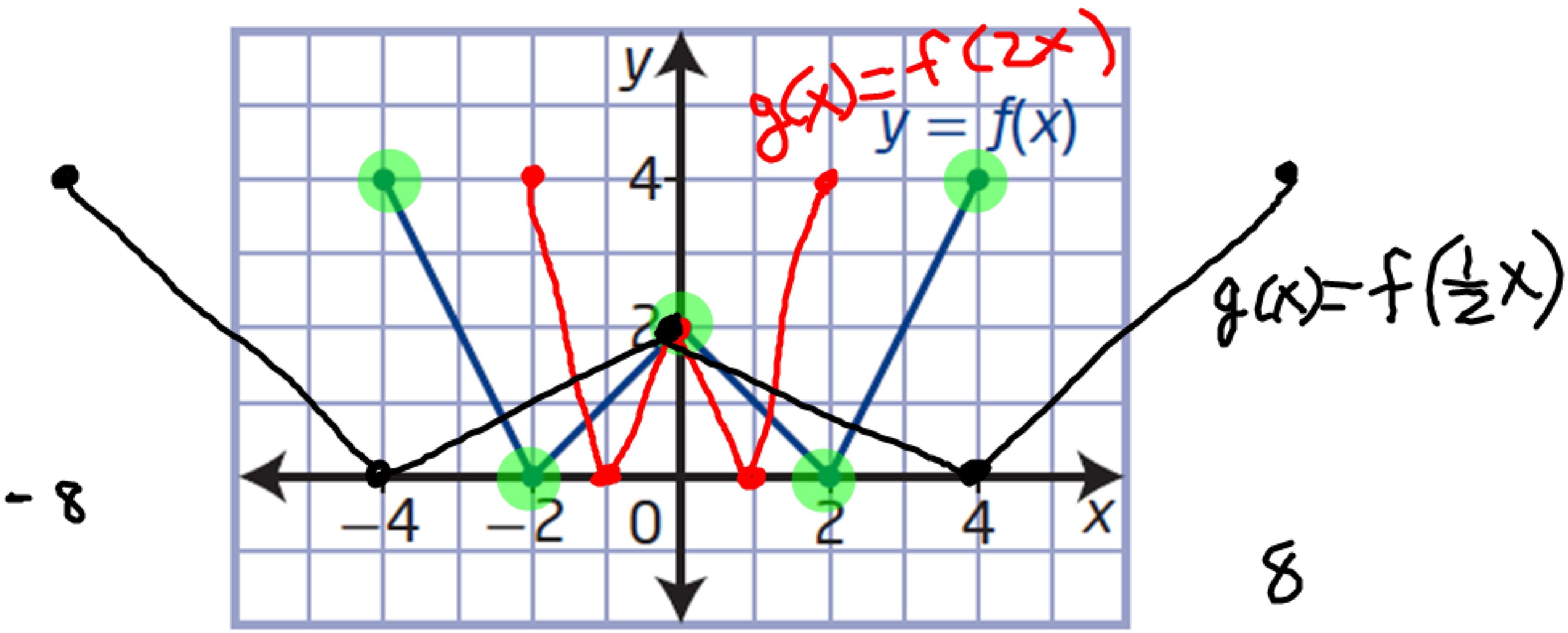
b) $g(x) = \frac{1}{2}f(x)$



Given $y = f(x)$, graph :

a) $g(x) = f(2x)$ → $\frac{1}{2}$

b) $g(x) = f(\frac{1}{2}x)$ → $\frac{1}{\frac{1}{2}} = 2$



vertical st. horizontal st.

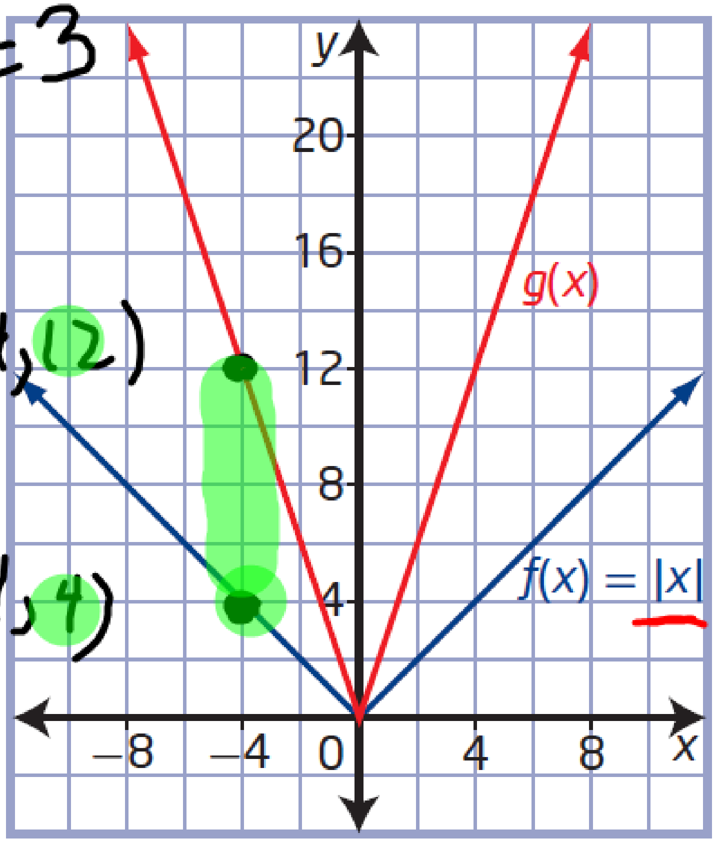
The graph *y=f(x)* has been transformed by a stretch. Write **two** possible equations for this graph.

Case 1

$a=3$

$(-4, 12)$

$(-4, 4)$

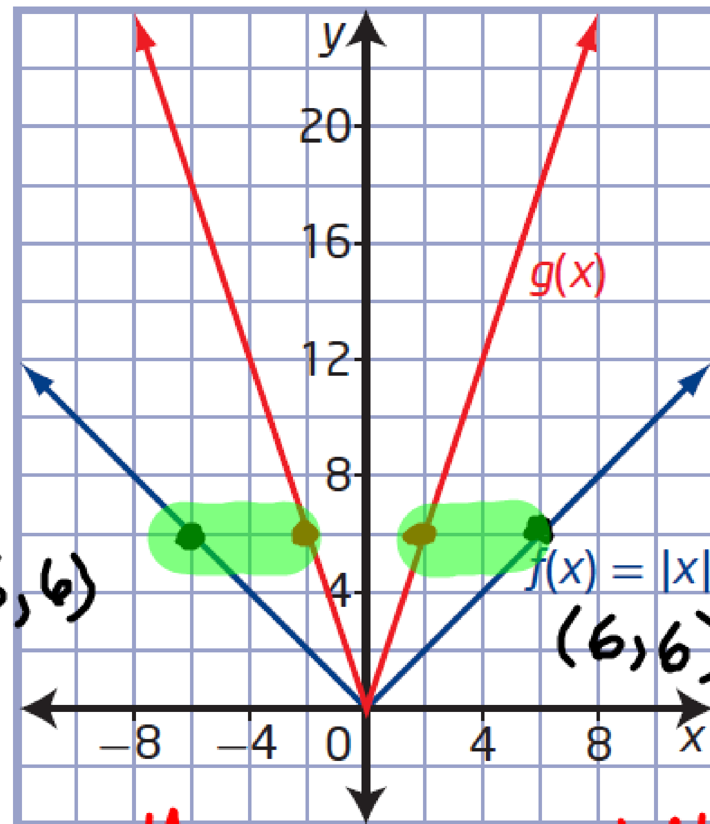


$g(x)=3|x|$

Case 2

$(-6, 6)$

$(6, 6)$

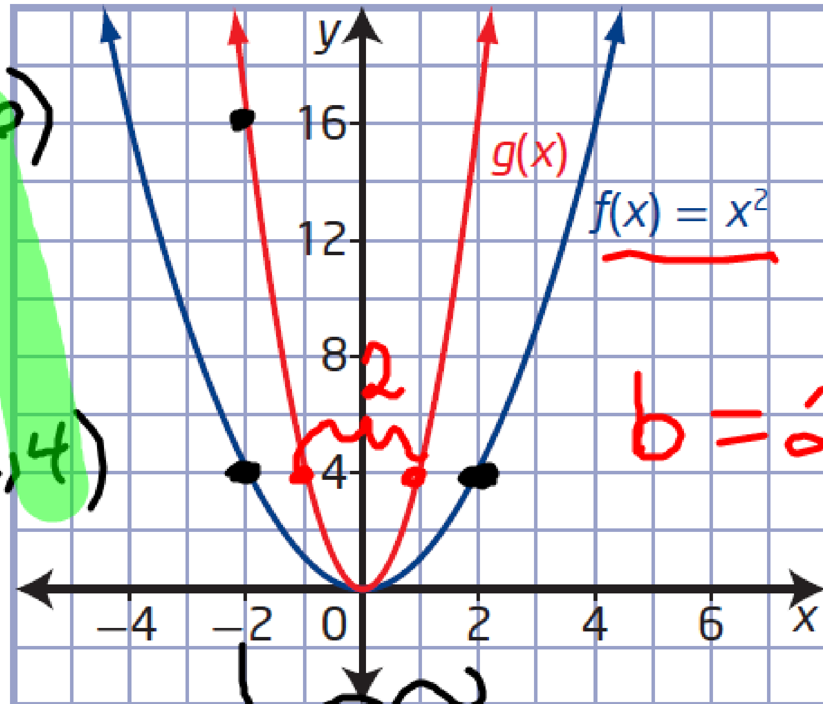


width 12 → width 4

$b=3$ $g(x)=|3x|$

The graph $y=x^2$ has been transformed. Write the equation for the graph $g(x)$. a) vertical st

b) horizontal st.



(-2, 16)

(-2, 4)

$a=4$

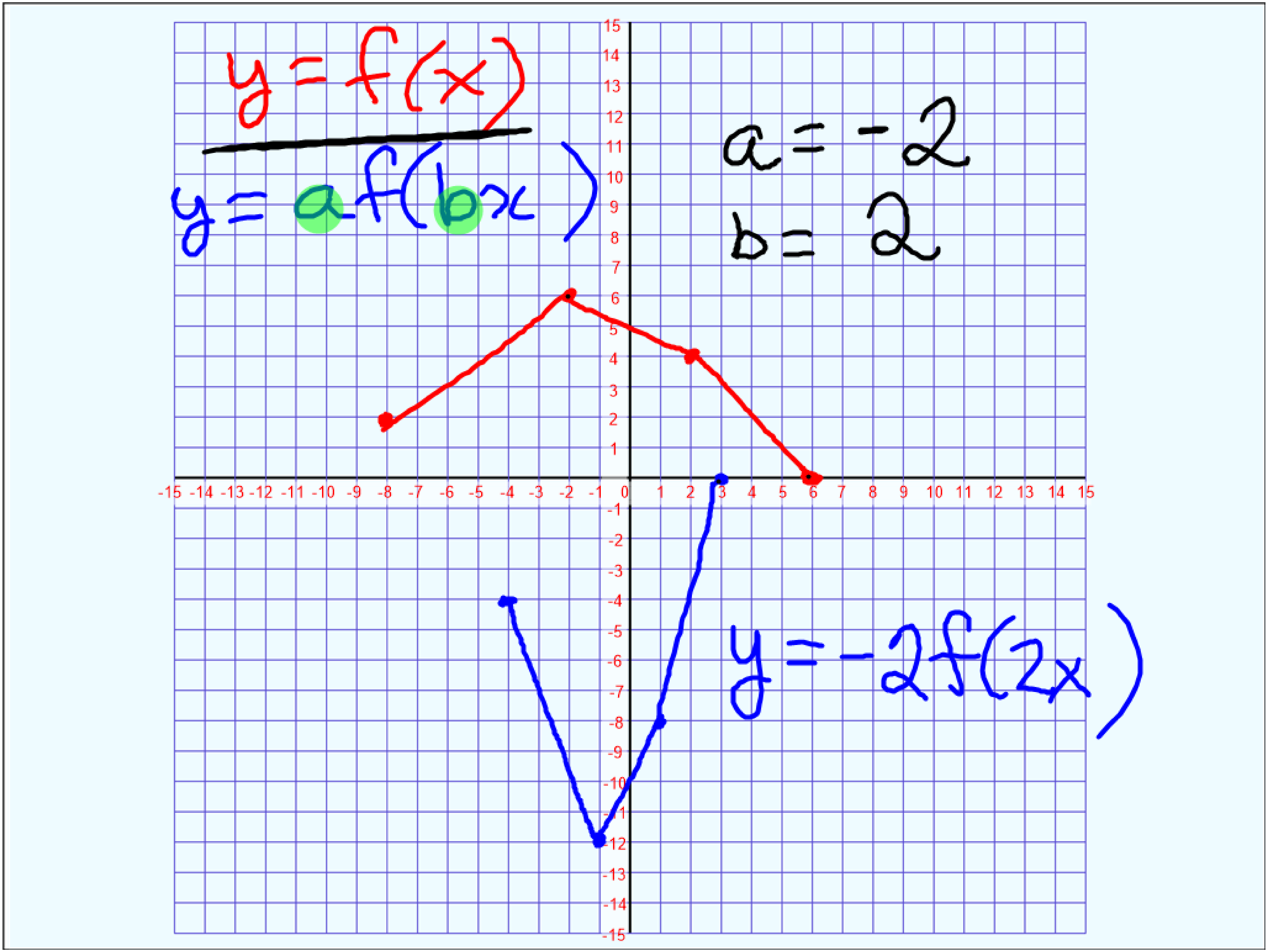
$b=2$

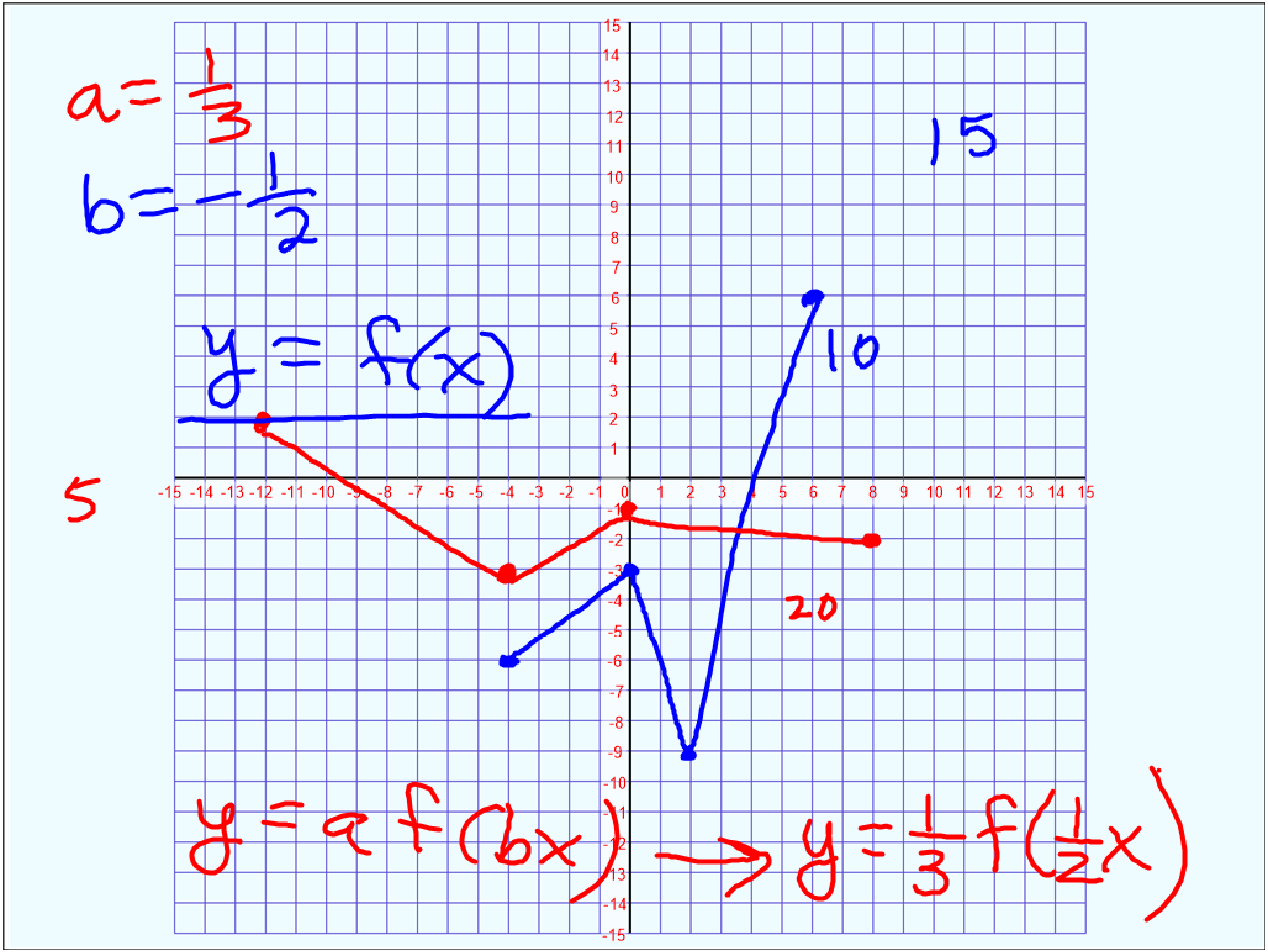
4

$g(x) = 4x^2$

$g(x) = (2x)^2$

$= 4x^2$


y = f(x)
y = af(bx)
a = -2
b = 2
y = -2f(2x)



$a = \frac{1}{3}$
$b = -\frac{1}{2}$
$y = f(x)$
15
10
5
20
$y = a f(bx) \rightarrow y = \frac{1}{3} f(\frac{1}{2}x)$

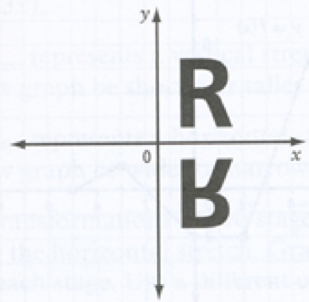
Key Ideas

KEY IDEAS

- A reflection creates a mirror image of the graph of a function across a line of reflection. Any points where the function crosses the line of reflection do not move (invariant points). A reflection may change the orientation of the function but its shape remains the same.

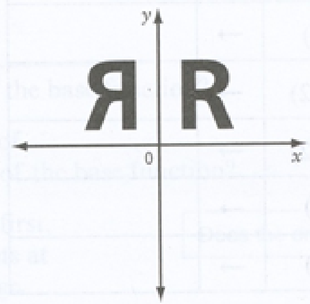
Vertical reflection:

- $y = -f(x)$
- $(x, y) \rightarrow (x, -y)$
- line of reflection: x-axis
- also known as a reflection in the x-axis



Horizontal reflection:

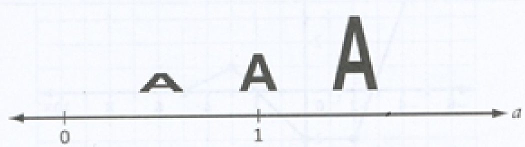
- $y = f(-x)$
- $(x, y) \rightarrow (-x, y)$
- line of reflection: y-axis
- also known as a reflection in the y-axis



- A stretch changes the shape of a graph but not its orientation. A vertical stretch makes a function shorter (compression) or taller (expansion) because the stretch multiplies or divides each y-coordinate by a constant factor while leaving the x-coordinate unchanged. A horizontal stretch makes a function narrower (compression) or wider (expansion) because the stretch multiplies or divides each x-coordinate by a constant factor while leaving the y-coordinate unchanged.

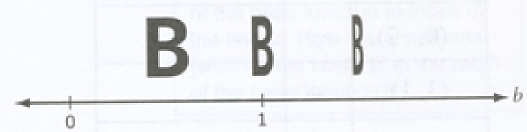
Vertical stretch by a factor of $|a|$:

- $y = af(x)$ or $\frac{1}{a}y = f(x)$
- $(x, y) \rightarrow (x, ay)$
- shorter: $0 < |a| < 1$
- taller: $|a| > 1$



Horizontal stretch by a factor of $\frac{1}{|b|}$:

- $y = f(bx)$
- $(x, y) \rightarrow \left(\frac{1}{b}x, y\right)$
- wider: $0 < |b| < 1$
- narrower: $|b| > 1$



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#’s 3,5,6,7,10,12,15,C3