1.2 Factoring

There are many math skills essential to Calculus, but perhaps none quite as important as being able to **factor** expressions.

In past math classes we have usually factored polynomials over the **set of rational numbers.**

$$x^2 - 5x - 14$$

$$= (x-7)(x+2)$$

$$\frac{a}{b} \quad b \neq 0$$

$$\sqrt{3}$$

Coefficients 1,-5,2 are all rational numbers!

The **product** of $\left(x-1-\sqrt{2}\right)\left(x-1+\sqrt{2}\right)$ results in

x^2-2x-1

Therefore x^2-2x-1 can be factored over the set of polynomials with real coefficients.

Coefficients $-1-\sqrt{2}$ and $-1+\sqrt{2}$ are real numbers

For convenience we write:

factor $P(x)$ over Q means to factor over the rationals

factor $P(x)$ over R means to factor over the reals

Factoring a **Difference of Squares**

Ex. 1 Factor the following completely over Q.

a) $x^2 - 100$

$= (x + 10)(x - 10)$

* b) $25 - 4k^2$

$= (5 + 2k)(5 - 2k)$

c) $r^{10} - \frac{1}{81}$

$= \left(r^5 + \frac{1}{9}\right)\left(r^5 - \frac{1}{9}\right)$

d) $-t^2 - 36$

$= -1(t^2 + 36)$

Ex. 2 Factor the following completely over Q.

a) $16x^4 - 1$

b) $a^8 - b^8$

$= (4x^2 + 1)(4x^2 - 1)$

$= (4x^2 + 1)(2x + 1)(2x - 1)$

b) $(a^4 + b^4)(a^4 - b^4)$

$= (a^4 + b^4)(a^2 + b^2)(a^2 - b^2)$

$= (a^4 + b^4)(a^2 + b^2)(a + b)(a - b)$

Ex. 3 Factor the following completely over Q.

$$(t-3)^2-25$$

$t^2-6t+9-25$

$=((t-3)+5)((t-3)-5)$

$=(t+2)(t-8)$

$=t^2-6t-16$

Ex. 4 Factor the following completely over Q.

$$9-(x+4)^2$$

$$= (3-(x+4))(3+(x+4))$$

$$= (3-x-4)(3+x+4)$$

$$= (-x-1)(x+7)$$

$$= -(x+1)(x+7)$$

Ex.5 Factor **over the *R***

$a)\ x^2 - 3$

$b)\ 12x^2 - 49$

$= (x + \sqrt{3})(x - \sqrt{3})$

$= (\sqrt{12}\,x - 7)(\sqrt{12}\,x + 7)$

$= (2\sqrt{3}\,x - 7)(2\sqrt{3}\,x + 7)$

Factoring Trinomials

Guess and Check

Note x^2 term has a coefficient equal to 1.

Ex.7 Factor the "simple" trinomial:

$$x^2 + 6x + 8$$

$$= (x+4)(x+2)$$

Ex.8 Factor the "simple" trinomial:

$$x^2 - 9xy + 14y^2$$

$$= (x - 2y)(x - 7y)$$

Decomposition

Rainbow Method 

Ex. 9 Factor the following trinomial:

Note x^2 term has a coefficient greater than 1.

$$9x^2 + 6x - 8$$

$P = -72$

$S = 6$

$12, -6$

$$\underbrace{9x^2 + 12x}\ \underbrace{-6x - 8}$$

$$= 3x(3x+4) - 2(3x+4)$$

$$= (3x+4)(3x-2)$$

$$3x^2 - 7x + 1$$

$$= a\,(x - 1^{st}\,z)(x - 2^{nd}\,z)$$

$$x = \frac{7 \pm \sqrt{49 - 12}}{2(3)}$$

$$= 3\left(x - \left(\frac{7 + \sqrt{37}}{6}\right)\right)\left(x - \left(\frac{7 - \sqrt{37}}{6}\right)\right)$$

$$x = \frac{7 \pm \sqrt{37}}{6}$$

$$x = \frac{7 + \sqrt{37}}{6} \quad \text{OR} \quad x = \frac{7 - \sqrt{37}}{6}$$

$3x^2 - 7x + 1$

$D = b^2 - 4ac$

$D = 37$

$x^2 - 6x + 9$

$(x-3)(x-3)$

Assignment
Page 16
#'s 3,4,7,

$25x^2 - 6$

$(5x - \sqrt{6})(5x + \sqrt{6})$

Factoring Sums and Differences of Cubes

An expression like:

$x^3 + y^3$ is called a sum of cubes.

$= (x+y)(x^2 - xy + y^2)$

PRIME

$x^3 - y^3$ is called a difference of cubes

$= (x-y)(x^2 + xy + y^2)$

Ex. 1 Factor the following completely

$$8a^3 - 125c^3$$

$$= (2a - 5c)(4a^2 + 10ac + 25c^2)$$

SOAP

S – Same

O – Opposite

A – Always

P - Positive

Ex. 2 Factor the following completely

$$\frac{1}{1000}b^3+1$$

$$=\left(\frac{1}{10}b+1\right)\left(\frac{1}{100}b^2-\frac{1}{10}b+1\right)$$

Ex.3 Factor $x^3 - 5$ over the *R*.

$$= \left(x - \sqrt[3]{5}\right)\left(x^2 + \sqrt[3]{5}\,x + \sqrt[3]{25}\right)$$

Common Factors and Related Issues

It is customary to begin factoring by removing the $\boxed{\text{GCF.}}$

$$x^5y^2 - 25x^3y^4$$

$$= x^3y^2(x^2 - 25y^2)$$

$$= x^3y^2(x-5y)(x+5y)$$

Ex.6 Factor the following so there are no negative exponents and fractional coefficients.

a) $x + 7 + 6x^{-1}$

$$= x^{-1}\underbrace{(x^2 + 7x + 6)}$$

$$= x^{-1}(x+6)(x+1)$$

$$\frac{x^1}{x^{-1}} \qquad \frac{7}{x^{-1}}$$

b) $\frac{3}{2}x^{3/2}+\frac{11}{4}x^{1/2}+x^{-1/2}$

$$=\frac{1}{4}x^{-1/2}\left(6x^2+11x+4\right)$$

$$=\frac{1}{4}x^{-1/2}(3x+4\quad)(2x+1\quad)$$

c) $2a^{7/2}b^{-1/2} - \frac{1}{4}a^{1/2}b^{5/2}$

$= \frac{1}{4}a^{1/2}b^{-1/2}\left(8a^3 - b^3\right)$

$= \frac{1}{4}a^{1/2}b^{-1/2}(2a-b)(4a^2+2ab+b^2)$

$$\frac{1}{5}x^{3/2}y^{-1} - \frac{7}{5}x^{1/2} + \frac{12}{5}x^{-1/2}y^{1}$$

$$= \frac{1}{5}x^{-1/2}y^{-1}\left(x^2 - 7xy + 12y^2\right)$$

$$= \frac{1}{5}x^{-1/2}y^{-1}(x-3y)(x-4y)$$

http://www.youtube.com/watch?v=hJY5Z13q2LY

Assignment

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#'s 9 b-d, 10 a,c,

14 b,d,f,h,i,l,o,p,r, 18

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#'s 5 a,b,c,d,e,i,k,n

Application

18. A cell phone manufacturer estimates that the selling price per phone, p, is related to the number of *millions* of cell phones manufactured, x, by the function $p(x) = 125 - 5x^2$. This is known as the **demand function** for the cell phones. If the company sells x cell phones, then the company's **revenue function**, $R(x)$, (amount of money taken in by the company) will be the product of x and $p(x)$. Thus the revenue function in *millions of dollars* is $R(x) = xp(x) = x(125 - 5x^2)$. If the actual cost to manufacture each cell phone is \$45, then the company's **cost function**, $C(x)$, in *millions of dollars* is given by $C(x) = 45x$.

(a) What was the selling price of each phone if 2 million were sold?
(b) What was the revenue from the sale of the phones in part (a)?
(c) What was the cost to produce the 2 million phones?
(d) The company's **profit function**, $P(x)$, is determined by finding the difference between the revenue and the cost. That is $P(x) = R(x) - C(x)$. What profit did the company make in selling 2 million phones?
(e) What is the profit function in factored form if x phones are sold?

a) $P(2) = 125 - 5(2)^2$
$= \$105$

b) $R(2) = 2(105) = \$210$ million

c) $C(2) = 45(2) = 90$ million

d) $P(2) = \$210\ 000\ 000 - 90\ 000\ 000$

$= \$120\ 000\ 000$

e) $P(x) = R(x) - C(x)$

$$\begin{aligned} P(x) &= x(125 - 5x^2) - 45x \\ &= 125x - 5x^3 - 45x \\ &= 80x - 5x^3 \\ &= 5x(16 - x^2) = 5x(4 - x)(4 + x) \end{aligned}$$