Unit 1 Transformations and Functions

Section 1.1 Horizontal and Vertical Transformations

In this section we will see how the graph of a new function can be obtained from the graph of an old function by adding some transformations.

A transformation of a function alters the equation and any combination of the location, shape and orientation of the graph.

Points on the original graph correspond to points on the transformed, or image, graph. The relationship between these sets of points can be called a **mapping**.

Mapping notation can be used to show a relationship between the coordinates of a set of points, (x, y), and the coordinates of a corresponding set of points, $(x, y + 3)$, for example, as $(x, y) \rightarrow (x, y + 3)$.

image point

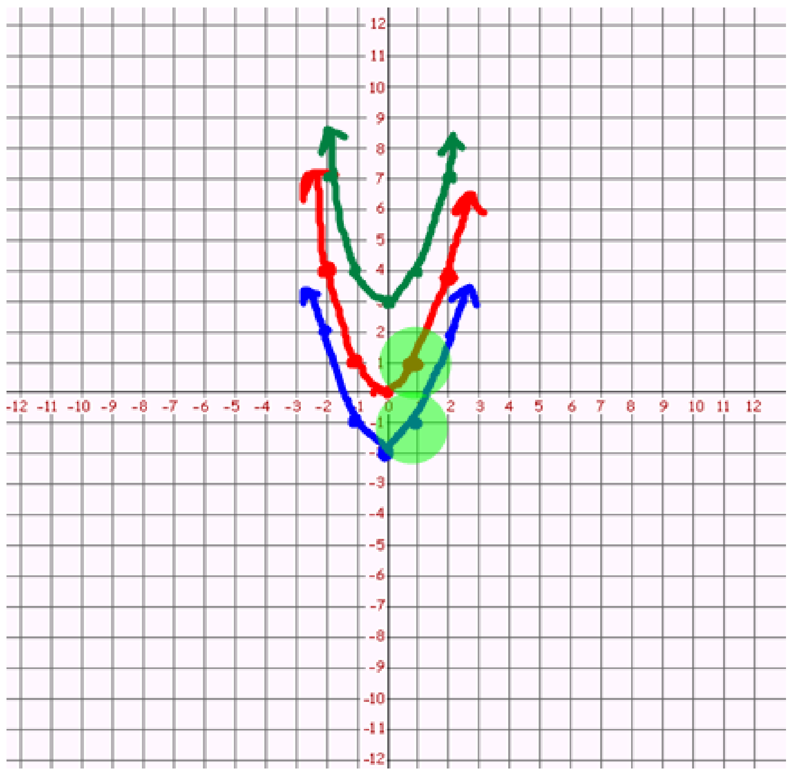
- the point that is the result of a transformation of a point on the original graph

Say we have the function $y = x^2$.

Now lets compare to $y = x^2 - 2$?

Now lets compare to $y = x^2 + 3$?

x	$y=x^2$
0	0
1	1
-1	1
2	4
-2	4



x	$y=x^2-2$
0	-2
1	-1
-1	-1
2	2
-2	2

What did we notice?

x	$y = x^2 + 3$
0	3
1	4
-1	4
2	7
-2	7

We call this type of transformation a **vertical translation.**

A **translation** occurs when the location of the graph changes but not its shape or orientation.

Therefore if our function is $y = f(x)$, then $y = f(x) + k$ vertically shifts the graph of $y = f(x)$ k units.

If $k > 0$ the graph shifts up.

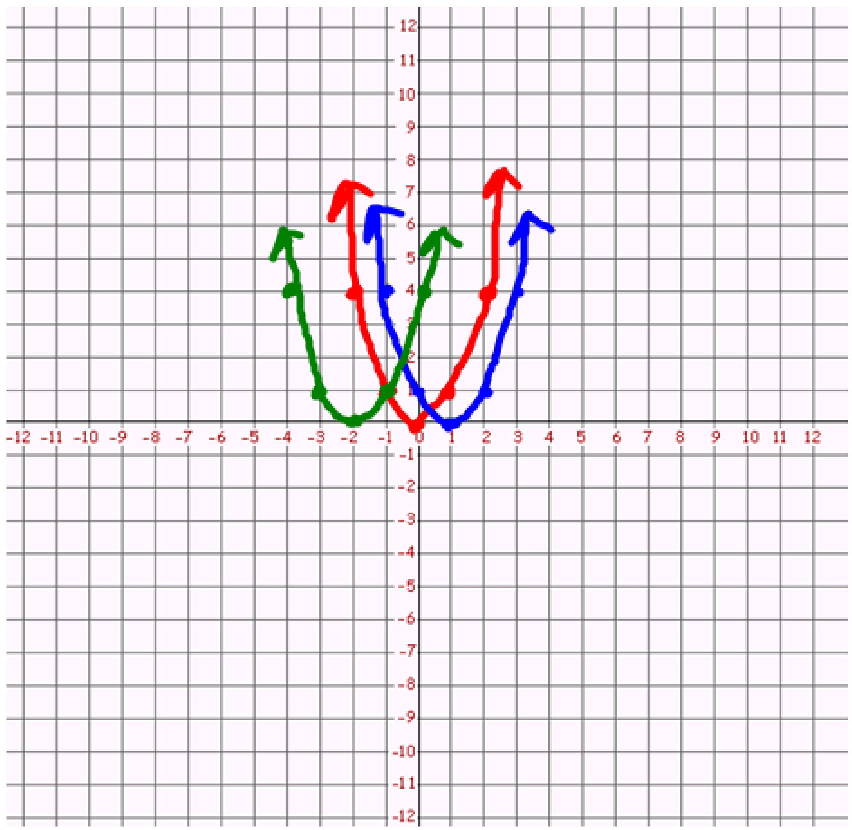
If $k < 0$ the graph shifts down.

Let's rexamine the function $y = x^2$.

Let's compare to the function $y = (x-1)^2$.

Let's compare to the function $y = (x+2)^2$.

x	$y = x^2$
0	0
1	1
-1	1
2	4
-2	4



x	$y = (x-1)^2$
1	0
2	1
0	1
3	4
-1	4

What did we notice?

x	$y=(x+2)^2$
-2	0
-1	1
-3	1
0	4
-4	4

We call this type of transformation a **horizontal translation.**

Therefore if our function is $y = f(x)$, then $\boxed{y = f(x - h)}$ horizontally shifts the graph of $y = f(x)$ h units.

If $h > 0$ the graph shifts to the right.

If $h < 0$ the graph shifts to the left.

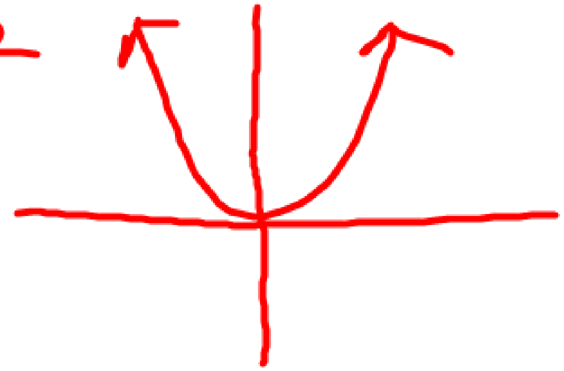
$y = f(x - 2)$ $h = 2$

$y = f(x + 2)$ $h = -2$

Your Turn

How do the graphs of $\boxed{y + 1 = x^2}$ and $\boxed{y = (x + 3)^2}$ compare to the graph of $y = x^2$? Justify your reasoning.

$y = x^2$



$y = x^2 - 1$

$K = -1$

shift graph 1 down

$y = (x+3)^2$

$h = -3$

shift graph 3 left.

$(x, y) \longrightarrow (x-5,\ y-2)$

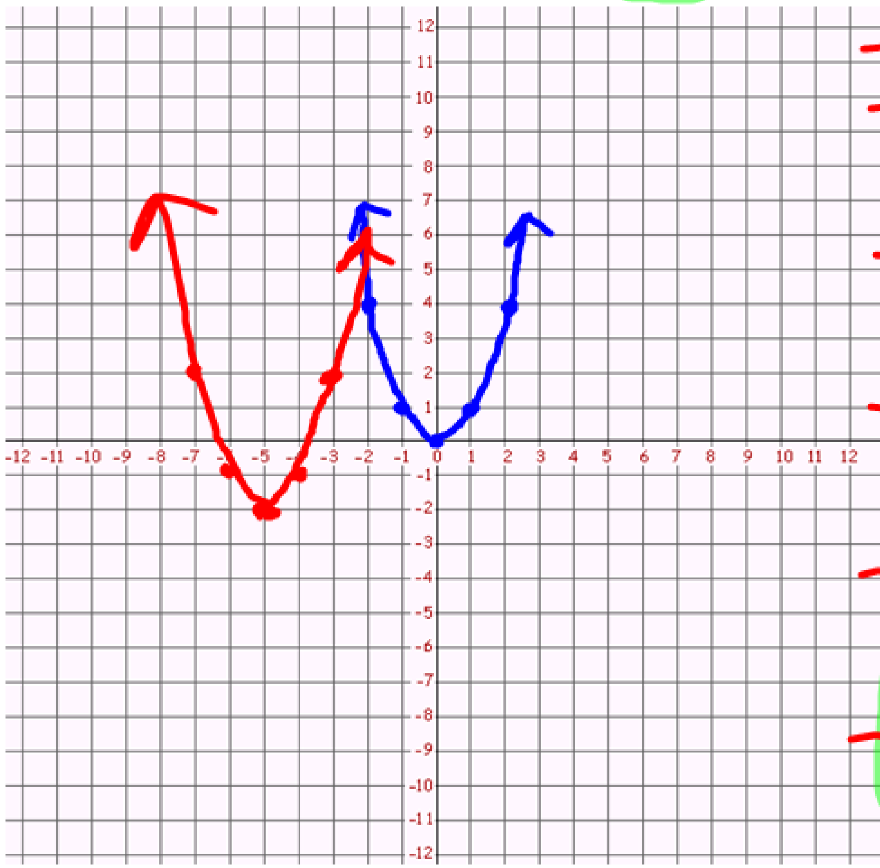
Your Turn #1

From your knowledge of the graph $y = x^2$, graph the following function:

$$y = (x+5)^2 - 2$$

$h = -5$
$k = -2$

x	$y = x^2$
0	0
1	1
-1	1
2	4
-2	4



x	$y = (x+5)^2 - 2$
-5	-2
-4	-1
-6	-1
-3	2
-7	2

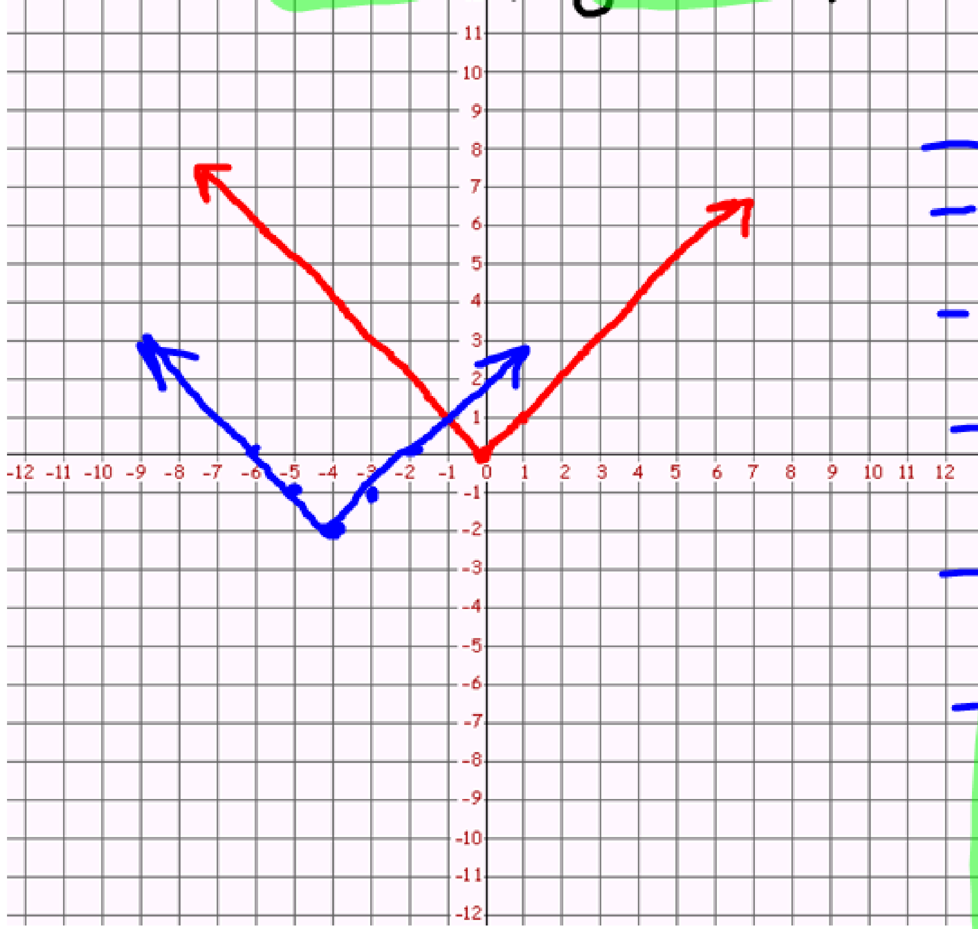
$h = -4 \quad k = -2$

Say we have the function $y = |x|$.

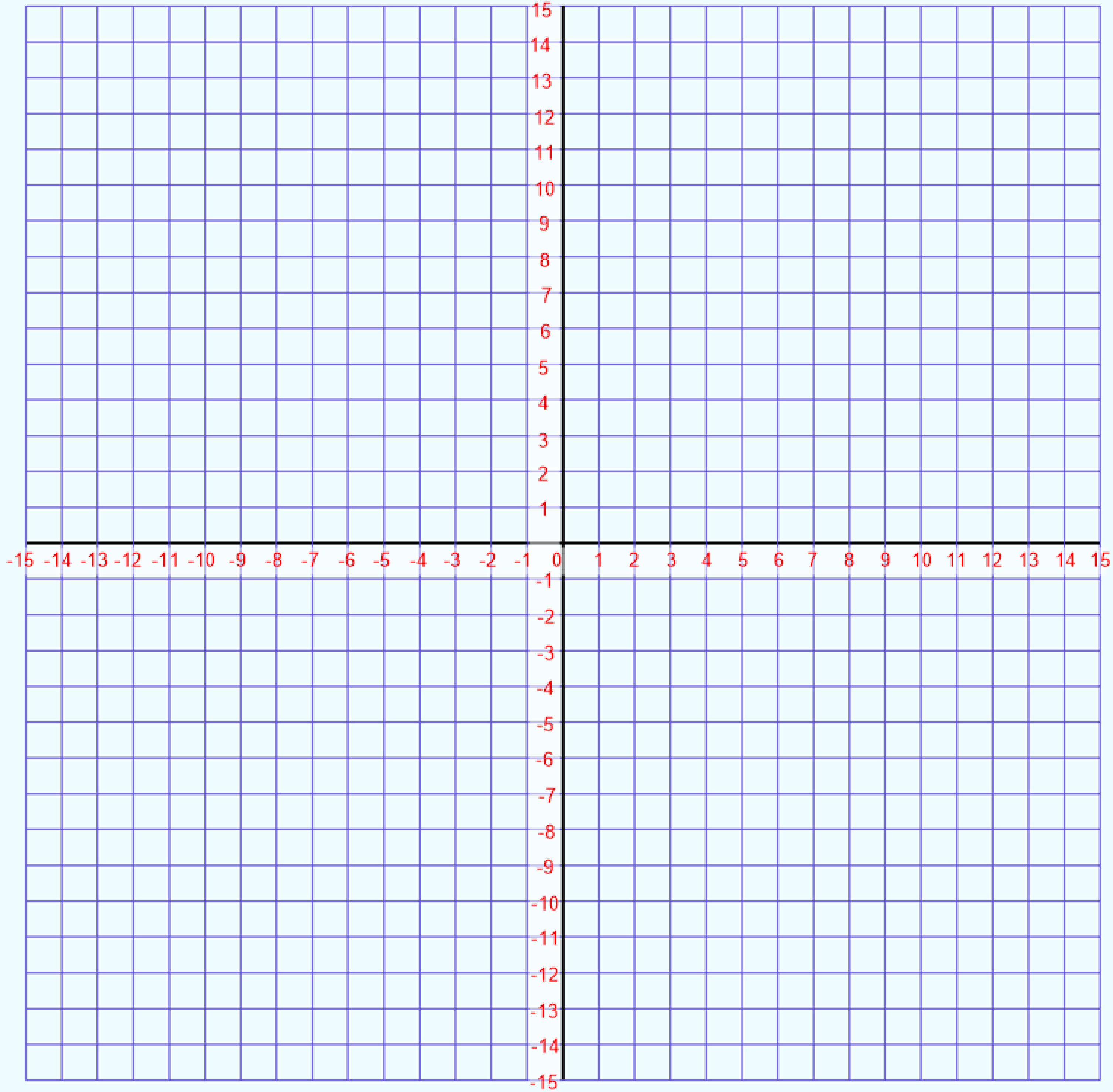
Now lets compare to $y = |x+4| - 2$?

$(x, y) \rightarrow (x-4, y-2) \rightarrow (x, y)$

x	$y=\|x\|$
0	0
1	1
-1	1
2	2
-2	2
3	3
-3	3



x	$y=\|x+4\|-2$
-4	-2
-3	-1
-5	-1
-2	0
-6	0

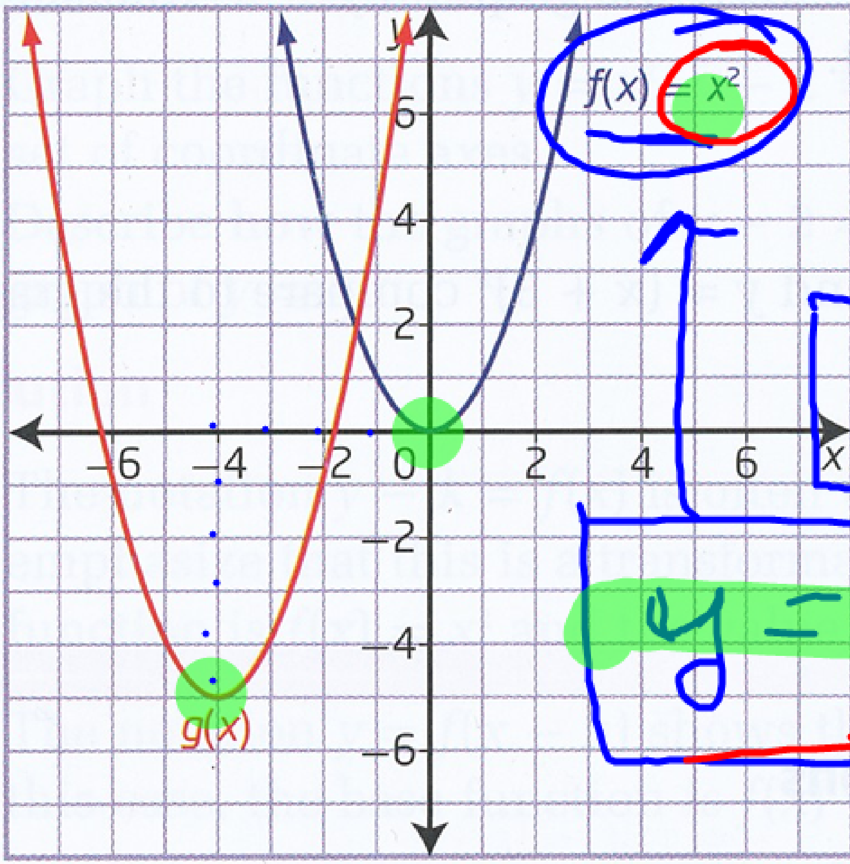

-15 -14 -13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 -1 -2 -3 -4 -5 -6 -7 -8 -9 -10 -11 -12 -13 -14 -15

Determine the Equation of a Translated Function

Describe the translation that has been applied to the graph of $f(x)$ to obtain the graph of $g(x)$. Determine the equation of the translated function in the form $y - k = f(x - h)$.

$h = -4 \quad k = -5$

a)



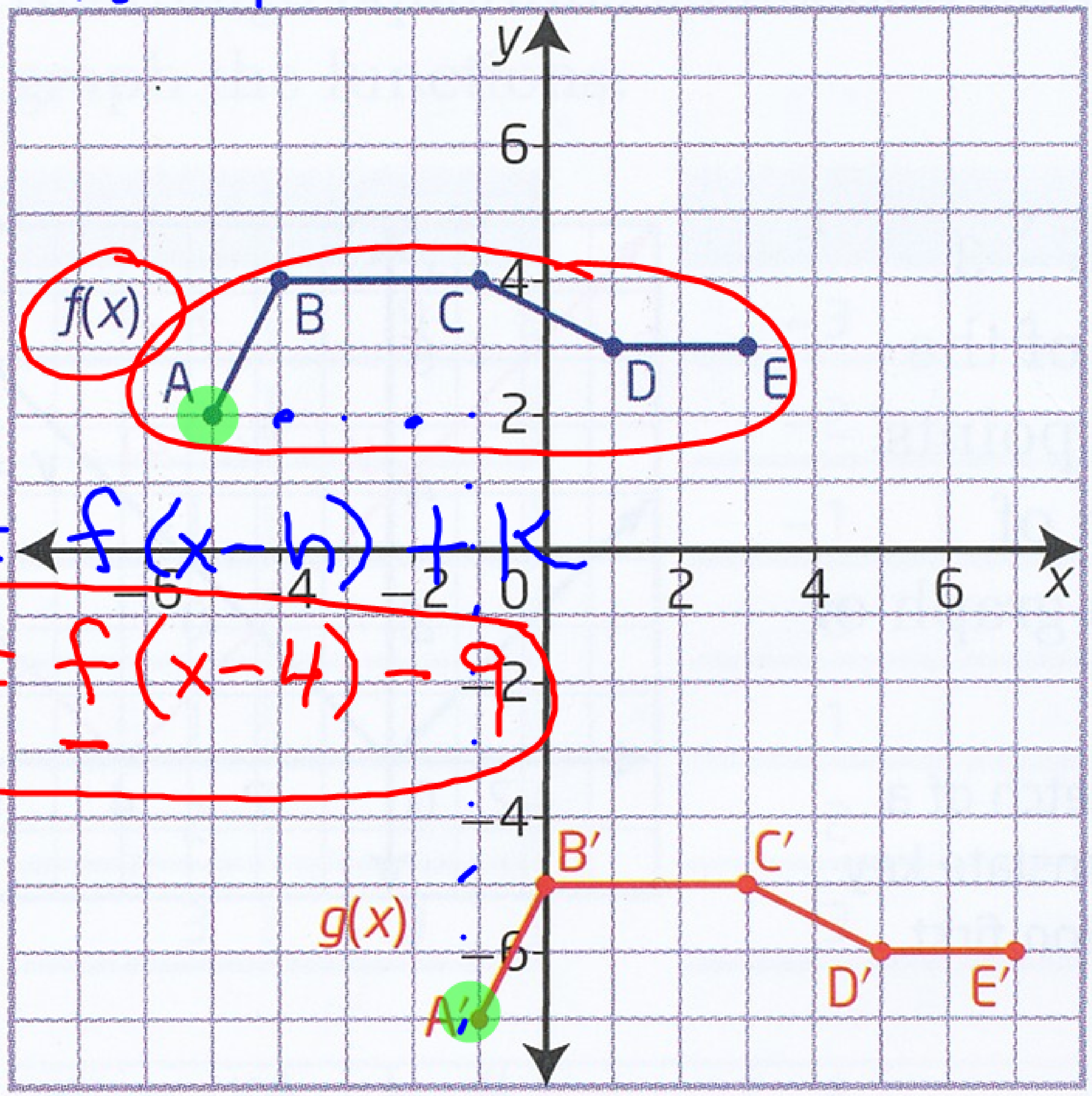
$y + 5 = f(x + 4)$

$\boxed{y = f(x+4) - 5}$

$\boxed{y = (x+4)^2 - 5}$

$h = 4 \quad k = -9$

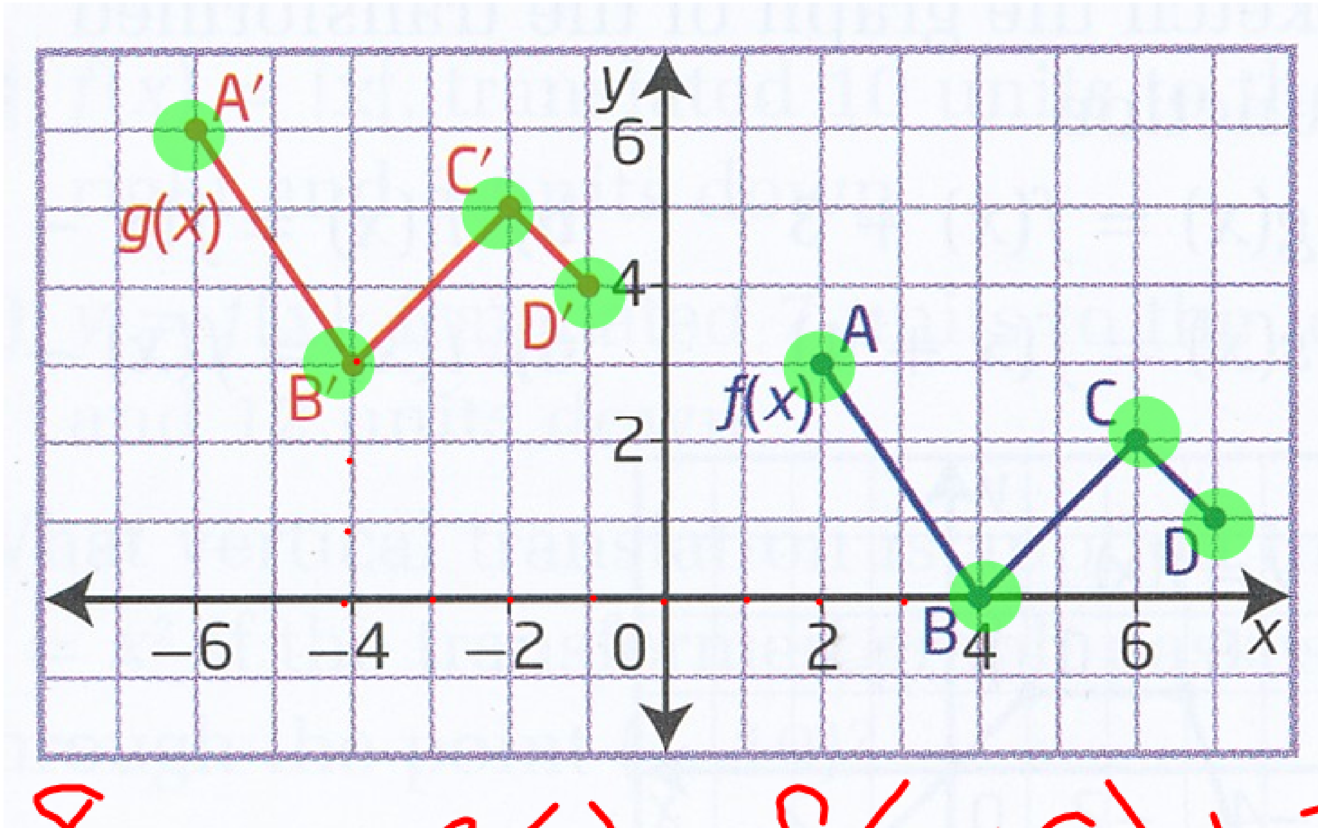
b)



$g(x) = f(x-h) + k$

$g(x) = f(x-4) - 9$

Your Turn #2



$h = -8$

$g(x) = f(x+8) + 3$

$k = 3$

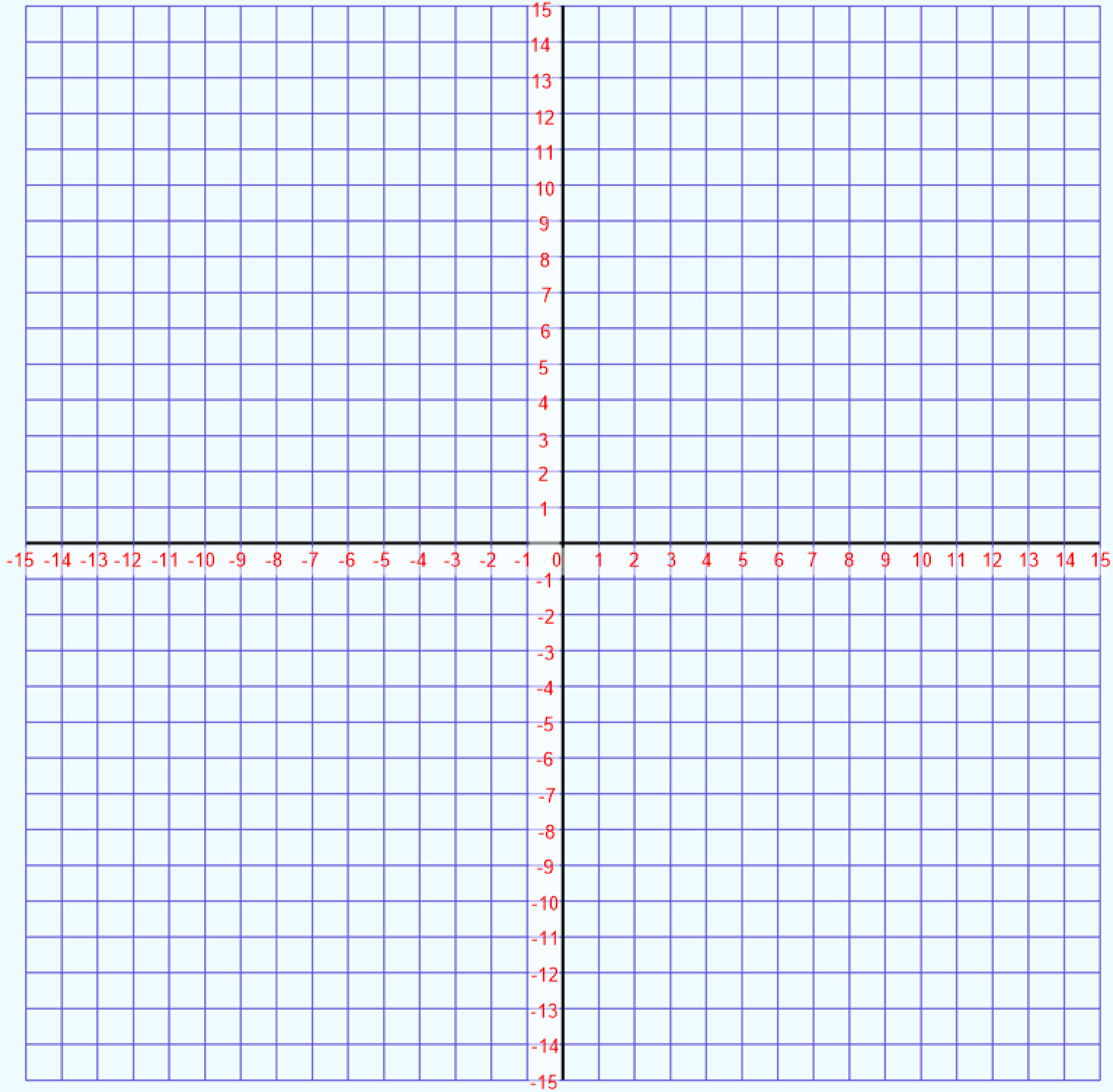
For a horizontal translation and a vertical translation where every point (x, y) on the graph of $y = f(x)$ is transformed to $(x + h, y + k)$, the equation of the transformed graph is of the form $y - k = f(x - h)$.

$y = f(x-h) + K$

See table page 12

Assignment Page 12

#'s 1,2,4,5,6,7,9,11,12,17,18


-15 -14 -13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 -1 -2 -3 -4 -5 -6 -7 -8 -9 -10 -11 -12 -13 -14 -15

18. Use translations to describe how the graph of $y = \frac{1}{x}$ compares to the graph of each function.

a) $y - 4 = \frac{1}{x}$

b) $y = \frac{1}{x + 2}$

c) $y - 3 = \frac{1}{x - 5}$

d) $y = \frac{1}{x + 3} - 4$